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ON PHONON HYDRODYNAMICS OF DIELECTRICS

Dielectric phonon hydrodynamics is constructed on the basis of our generalization of the Chapman–Enskog method for the case of the presence of non-hydrodynamic relaxation processes in a system. Phonon quasi-momentum density relaxation related to umklapp processes in phonon collisions is taken into account. The research is based on the Bogolyubov idea of the functional hypothesis. For the first time such relaxation process is investigated at the last stage with going beyond the limits of linearization and introducing a new small parameter into the theory. The proposed approach generalizes the known Grad's theory of relaxation, the main drawback of which is the absence of a small parameter in it. The built phonon hydrodynamics is not based on the local equilibrium assumption and allows investigating the domain of its applicability. The absence of the local equilibrium does not allow using the Landau definition of the phonon drift velocity. Therefore, here for the first time the relaxation process in the system is described by the phonon momentum density as a hydrodynamic variable. The obtained results are investigated in more detail for the case of moderately low temperatures. The steady states of the system are also considered.

Keywords: phonons hydrodynamics of dielectrics, non-hydrodynamic relaxation phenomena, local equilibrium violation, Chapman–Enskog method, moderately low temperatures, steady states.

1. Introduction

Kinetics of the phonon subsystem of dielectrics has a long research history starting from the pioneering paper by Akhiezer [1]. The main problem is to define the drift velocity of the phonon system and to describe a non-hydrodynamic relaxation of this velocity. In our paper [2] it was shown that the local equilibrium assumption is not valid in the presence of non-hydrodynamic relaxation. This means that the local Planck distribution with a velocity (see formula (5)) is not the main contribution to the non-equilibrium phonon distribution function. Therefore, this Planck distribution cannot be used as a definition of the drift velocity. In the present paper we propose using the phonon quasi-momentum density instead of the drift velocity as a hydrodynamic variable (this idea was presented in [3]). Non-hydrodynamic relaxation investigation is complicated because of the lack of a small parameter in this theory [4]. The well-known example of a theory related to this problem is given by the Grad theory of the Maxwell relaxation [5]. To simplify investigation of the non-hydrodynamic relaxation, we proposed in [6] studying the problem near the end of the relaxation process that introduces a new small parameter. Its physical meaning can be justified by comparing our relaxation theory in linear and quadratic approximations [7]. Here we limit ourselves by investigating the phonon hydrodynamics in dielectrics with the cubic symmetry of the classes O, O_{h}, and T_{h}. The paper is organized as follows. Section 2 formulates the basic equation of the proposed theory, introduces small parameters of the theory, and discusses the local equilibrium assumption. Section 3 builds phonon hydrodynamic equations in the perturbation theory. Section 4 discusses the developed theory at moderately low temperatures.

2. Basic equations of the theory

The investigation is based on the kinetic equation for phonons

$$\frac{\partial \mathrm{f}_{\alpha p}(x,t)}{\partial t} = -\frac{\partial \varepsilon_{\alpha p}}{\partial p_n}\frac{\partial \mathrm{f}_{\alpha p}(x,t)}{\partial x_n} + I^N_{\alpha p}(\mathrm{f}(x,t)) + I^U_{\alpha p}(\mathrm{f}(x,t)). \tag{1}$$

The main contribution to the collision integral corresponds to three-phonon processes. In equation (1) the collision integral consists of two parts: normal processes $I^N_{\alpha p}(\mathrm{f})$ and umklapp processes $I^U_{\alpha p}(\mathrm{f})$

$$I^N_{\alpha p}(\mathrm{f}) = \sum_{\alpha_1\alpha_2\alpha_3}\int_B d^3p_1 d^3p_2 d^3p_3 \,|\,\Phi(12,3)\,|^2\, \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3)\delta(p_1+p_2-p_3)\times$$
$$\times\{\delta_{\alpha\alpha_3}\delta(p-p_3) - \delta_{\alpha\alpha_1}\delta(p-p_1) - \delta_{\alpha\alpha_2}\delta(p-p_2)\}\{\mathrm{f}_1\mathrm{f}_2(1+\mathrm{f}_3) - (1+\mathrm{f}_1)(1+\mathrm{f}_2)\mathrm{f}_3\} \tag{2}$$

$$I^U_{\alpha p}(\mathrm{f}) = \sum_{\alpha_1\alpha_2\alpha_3}\int_B d^3p_1 d^3p_2 d^3p_3 \,|\,\Phi(12,3)\,|^2\, \delta(\varepsilon_1+\varepsilon_2-\varepsilon_3)\sum_{n\neq 0}\delta(p_1+p_2-p_3-\hbar b_n)\times$$
$$\times\{\delta_{\alpha\alpha_3}\delta(p-p_3) - \delta_{\alpha\alpha_1}\delta(p-p_1) - \delta_{\alpha\alpha_2}\delta(p-p_2)\}\{\mathrm{f}_1\mathrm{f}_2(1+\mathrm{f}_3) - (1+\mathrm{f}_1)(1+\mathrm{f}_2)\mathrm{f}_3\}$$

Here we use abbreviated notations $\varepsilon_{\alpha_i p_i} \equiv \varepsilon_i$, $\mathrm{f}_{\alpha_i p_i} \equiv \mathrm{f}_i$, $\Phi(\alpha_1 p_1;\alpha_2 p_2,\alpha_3 p_3) \equiv \Phi(12,3)$; α and p_n are phonon polarization and quasi-momentum, b_n is reciprocal lattice vector; the integrals are taken over the base cell of the reciprocal lattice B (for the simplicity below the term "momentum" is used instead of "quasi-momentum").

The equations of phonon hydrodynamics are a consequence of the law of conservation of energy and the law of change of the momentum of the system

$$\frac{\partial \varepsilon(x,t)}{\partial t} = -\frac{\partial q_l(x,t)}{\partial x_l}, \qquad \frac{\partial \pi_n(x,t)}{\partial t} = -\frac{\partial t_{nl}(x,t)}{\partial x_l} + R_n(x,t) \tag{3}$$

Densities of energy and momentum of phonons $\varepsilon(x,t)$, $\pi_n(x,t)$, the corresponding flux densities $q_l(x,t)$, $t_{nl}(x,t)$, and friction force density $R_n(x,t)$ are given by the formulas

$$\varepsilon(x,t) \equiv \sum_\alpha\int_B d\tau_p\, \varepsilon_{\alpha p}\mathrm{f}_{\alpha p}(x,t), \qquad \pi_n(x,t) \equiv \sum_\alpha\int_B d\tau_p\, p_n\mathrm{f}_{\alpha p}(x,t),$$
$$q_l(x,t) \equiv \sum_\alpha\int_B d\tau_p\, \varepsilon_{\alpha p}\frac{\partial \varepsilon_{\alpha p}}{\partial p_l}\mathrm{f}_{\alpha p}(x,t), \qquad t_{nl}(x,t) \equiv \sum_\alpha\int_B d\tau_p\, p_n\frac{\partial \varepsilon_{\alpha \mathrm{p}}}{\partial p_l}\mathrm{f}_{\alpha p}(x,t), \tag{4}$$
$$R_n(x,t) \equiv \sum_\alpha\int_B d\tau_p\, p_n I^U_{\alpha p}(\mathrm{f}(x,t))$$

($d\tau_p \equiv d^3p\,/\,(2\pi\hbar)^3$).

For temperatures $T \ll T_D$ (T_D is the Debye temperature) the phonon hydrodynamics construction is usually based on the assumption that for times $t \gg \tau_{\mathrm{fp}}$ (τ_{fp} is phonon-phonon free path time)

$$\mathrm{f}_{\alpha p}(x,t) \simeq n_{\alpha p}(\xi(x,t)), \qquad n_{\alpha p}(\xi) \equiv [e^{\frac{\varepsilon_{\alpha p} - p_n \upsilon_n}{T}} - 1]^{-1}, \qquad (\xi_\mu : T\ ,\ \upsilon_n) \tag{5}$$

where T, υ_n are temperature and macroscopic (drift) velocity of the phonon system. This is allegedly true because the main role in the system evolution belongs to the normal processes and then $n_{\alpha p}(\xi)$ is a corresponding equilibrium solution $I^N_{\alpha p}(n(\xi)) = 0$. This is called the local equilibrium assumption and this is the definition of the temperature T and drift velocity υ_n of the phonon system at the same time. However, distribution $n_{\alpha p}(\xi)$ at $\upsilon_n \neq 0$ is not a solution of the kinetic equation because $I^U_{\alpha p}(n(\xi)) \neq 0$. Therefore, a state described by $n_{\alpha p}(\xi)$ can not be the result of the natural evolution of the system. Further steps in the usual construction of phonon hydrodynamics is search for the distribution function $\mathrm{f}_{\alpha p}(x,t)$ in the form

$$\mathrm{f}_{\alpha p}(x,t) = n_{\alpha p}(\xi(x,t)) + \delta \mathrm{f}_{\alpha p}(x,t) \tag{6}$$

where $\delta\mathrm{f}_{\alpha p}(x,t)$ is a small correction in gradients of variables $\xi_\mu(x,t)$ (see, for example, [7]). This approach is in a contradiction with ideas of Hilbert, Chapman–Enskog, and Bogolyubov. It can be considered as an investigation of states in the vicinity of the local equilibrium state. Hereby the system must be placed in this state artificially.

In this article we build phonon hydrodynamics with a consequent description of relaxation process related to the drift velocity attenuation. The drift velocity definition beyond the local equilibrium assumption is a problem. Therefore, in this work we use the momentum density of the phonon system $\pi_n(x,t)$ as a hydrodynamic variable instead of the drift velocity υ_n.

In the considered system the momentum density attenuation is observed

$$\pi_l(x,t) \xrightarrow[t \gg \tau_\pi]{} 0 \tag{7}$$

because of umklapp processes. The relaxation process (7) can be observed in a spatially uniform state of the phonon system too. Its state is described by a distribution function $\mathrm{f}^{(0)}_{\alpha p}$ that should be considered as the main contribution in small gradients of variables $\xi_\mu(x,t)$ to the hydrodynamic distribution function $\mathrm{f}_{\alpha p}$. The function $\mathrm{f}^{(0)}_{\alpha p}$ cannot be calculated with the standard Chapman–Enskog method. Here $\mathrm{f}^{(0)}_{\alpha p}$ is calculated at the end of the relaxation process with introducing a new small parameter λ by the estimation

$$\pi_l(x,t) \sim \lambda^1 \qquad (\lambda \ll 1). \tag{8}$$

Study of hydrodynamic states is usually based on the Chapman–Enskog method for solving kinetic equations. In the general formulation of this method a particle distribution function $\mathrm{f}_{\alpha p}(x,t)$ (α – system component number, p_n – particle momentum) is considered as a functional $\mathrm{f}_{\alpha p}(x,\xi(t))$ of parameters $\xi_\mu(x,t)$ describing the hydrodynamic state. This state is observed at times much greater than the free path time τ_{fp}

$$\mathrm{f}_{\alpha p}(x,t)\xrightarrow[t\gg\tau_{\mathrm{fp}}]{}\mathrm{f}_{\alpha p}(x,\xi(t))\,. \tag{9}$$

The relation (9) is called the functional hypothesis ($\mathrm{f}_{\alpha p}(x,\xi)$ is a functional of $\xi_\mu(x)$) and a natural assumption that allows us to obtain a closed system of equations for the parameters $\xi_\mu(x,t)$ and express the observed quantities through these parameters.

Equations of hydrodynamics for variables $\xi_\mu(x,t)$ have a structure

$$\frac{\partial\xi_\mu(x,t)}{\partial t}=M_\mu(x,\mathrm{f}(\xi(t)))\,, \qquad \left(\frac{\partial^s\xi_\mu(x,t)}{\partial x_{n_1}...\partial x_{n_s}}\sim g^s,\quad g\ll 1\right) \tag{10}$$

The function $M_\mu(x,\mathrm{f}(\xi))$ can be expressed in terms of the distribution function $\mathrm{f}_{\alpha p}(x,\xi)$ by relations (3) and (4). The estimate in (10) introduces a standard hydrodynamic small parameter $g=l_{\mathrm{fp}}/L$ (l_{fp} is the free path length, L is a characteristic length on which the quantities $\xi_\mu(x,t)$ are substantially changed).

According to (1), (2), and (7), the functional $\mathrm{f}_{\alpha p}(x,\xi)$ satisfies the integro-differential equation

$$\sum_\mu\int d^3x'\frac{\delta\mathrm{f}_{\alpha p}(x,\xi)}{\delta\xi_\mu(x')}M_\mu(x',\mathrm{f}(\xi))=-\frac{\partial\varepsilon_{\alpha p}}{\partial p_n}\frac{\partial\mathrm{f}_{\alpha p}(x,\xi)}{\partial x_n}+I_{\alpha p}(\mathrm{f}(x,\xi)) \tag{11}$$

and additional conditions

$$\sum_\alpha\int_B d\tau_p\,\varepsilon_{\alpha p}\mathrm{f}_{\alpha p}(x,\xi)=\varepsilon(T(x))\,, \qquad \sum_\alpha\int_B d\tau_p\,p_n\mathrm{f}_{\alpha p}(x,\xi)=\pi_n(x)\,, \tag{12}$$

which define parameters $\xi_\mu(x)$ (the function $\varepsilon(T)$ is chosen below). Equation (11) is a nonlinear one and must be solved only approximately in the perturbation theory in small parameters g and λ. The physical meaning of the parameter λ can be justified by comparing linear and quadratic in λ contributions [7].

3. Phonon hydrodynamics

Equations (11) and (12) are solved in the double perturbation theory in terms of gradients (small parameter g) (10) and momentum density (small parameter λ) (8)

$$\mathrm{f}_{\alpha p}(x,\xi)=\mathrm{f}^{(0)}_{\alpha p}+\mathrm{f}^{(1)}_{\alpha p}+O(g^2)\,, \qquad \mathrm{f}^{(s)}_{\alpha p}=\mathrm{f}^{(s,0)}_{\alpha p}+\mathrm{f}^{(s,1)}_{\alpha p}+O(g^s\lambda^2) \tag{13}$$

(hereafter $A^{(m,s)}$ denotes contribution to a quantity A of the order $g^m\lambda^s$). Therefore, our calculation is based on estimates

$$\pi_l\sim\lambda^1\,,\quad \frac{\partial T}{\partial x_n}\sim g^1\lambda^0\,,\quad \frac{\partial\pi_l}{\partial x_n}\sim g^1\lambda^1\,,\quad T\sim g^0\lambda^0\,. \tag{14}$$

The first contributions to the hydrodynamic distribution function $\mathrm{f}_{\alpha p}(x,\xi)$ have the structure

$$\mathrm{f}_{\alpha p}^{(0)} = \overset{o}{n}_{\alpha p}\,, \qquad \overset{o}{n}_{\alpha p} \equiv [e^{\frac{\varepsilon_{\alpha p}}{T}} - 1]^{-1}\,; \qquad \mathrm{f}_{\alpha p}^{(0,1)} = \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})A_{\alpha n}(p)\pi_n\,,$$

$$\mathrm{f}_{\alpha p}^{(0,2)} = \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})B_{\alpha nl}(p)\pi_n\pi_l\,, \qquad \mathrm{f}_{\alpha p}^{(1,0)} = \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})C_{\alpha n}(p)\frac{\partial T}{\partial x_n}\,, \tag{15}$$

$$\mathrm{f}_{\alpha p}^{(1,1)} = \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})\left\{D_{\alpha nl}(p)\frac{\partial \pi_n}{\partial x_l} + E_{\alpha nl}(p)\frac{\partial T}{\partial x_n}\pi_l\right\}.$$

According to additional condition (12), the temperature is defined and expressed through the energy density by the formula

$$\sum_{\alpha}\int_B d\tau_p\, \varepsilon_{\alpha p}\, \overset{o}{n}_{\alpha p} = \varepsilon(T)\,.$$

Integral equations and additional conditions of the generalized by us Chapman–Enskog method are: for the function $A_{\alpha n}(p)$

$$\nu A_{\alpha n}(p) = \sum_{\alpha'}\int d^3p' K_{\alpha\alpha'}(p,p')A_{\alpha' n}(p')\,, \qquad \langle A_{\alpha n}(p)p_n\rangle = 3\,; \tag{16}$$

for the function $B_{\alpha nl}(p)$

$$2\nu B_{\alpha nl}(p) - \frac{1}{2}\sum_{\alpha'\alpha''}\int d^3p' d^3p'' K_{\alpha,\alpha'\alpha''}(p,p'p'')A_{\alpha' n}(p')A_{\alpha'' l}(p'') =$$
$$= \sum_{\alpha'}\int d^3p' K_{\alpha\alpha'}(p,p')B_{\alpha' nl}(p')\,, \qquad \langle B_{\alpha nl}(p)\varepsilon_{\alpha p}\rangle = 0\,; \tag{17}$$

for the function $C_{\alpha n}(p)$

$$A_{\alpha n}(p)\left(\frac{\partial P}{\partial T} + \nu_T\right) - \frac{\varepsilon_{\alpha p}}{T^2}\frac{\partial \varepsilon_{\alpha p}}{\partial p_n} = \sum_{\alpha'}\int d^3p' K_{\alpha\alpha'}(p,p')C_{\alpha' n}(p')\,, \qquad \langle C_{\alpha n}(p)p_n\rangle = 0\,; \tag{18}$$

and for the function $D_{\alpha nn}(p)$

$$\frac{h}{cT^2}\varepsilon_{\alpha p}\delta_{nl} - \frac{\partial \varepsilon_{\alpha p}}{\partial p_l}A_{\alpha n}(p) + \nu D_{\alpha nl}(p) = \sum_{\alpha'}\int_B d^3p' K_{\alpha\alpha'}(p,p')D_{\alpha' nl}(p')\,,$$
$$\langle \varepsilon_{\alpha p} D_{\alpha nn}(p)\rangle = 0 \tag{19}$$

where heat capacity c and phonon gas pressure P

$$P = \sum_{\alpha}\frac{1}{3h^3}\int d^3p\, \overset{o}{n}_{\alpha p}\, p_n \frac{\partial \varepsilon_{\alpha p}}{\partial p_n}\,, \qquad c = \frac{\partial \varepsilon}{\partial T} \tag{20}$$

are introduced. Function $E_{\alpha nl}(p)$ in (15) describes crossover phenomena and will be discussed in another paper.

The kernel of the integral equations (16)-(19) $K_{\alpha\alpha'}(p,p')$, special average $\langle g_{\alpha p}\rangle$, and necessary below bilinear form $\{g_{\alpha p},h_{\alpha p}\}$ are defined by formulas

$$I_{\alpha p}(\overset{o}{n}+\delta \mathrm{f})\equiv\sum_{\alpha'}\int d^3p' M_{\alpha\alpha'}(p,p')\delta \mathrm{f}_{\alpha' p'}+\frac{1}{2}\sum_{\alpha'\alpha''}\int d^3p'd^3p'' M_{\alpha,\alpha'\alpha''}(p,p'p'')\delta \mathrm{f}_{\alpha' p'}\delta \mathrm{f}_{\alpha'' p''}$$

$$M_{\alpha\alpha'}(p,p')\overset{o}{n}_{\alpha' p'}(1+\overset{o}{n}_{\alpha' p'})\equiv-\overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})K_{\alpha\alpha'}(p,p'),$$

$$M_{\alpha,\alpha'\alpha''}(p,p'p'')\overset{o}{n}_{\alpha' p'}(1+\overset{o}{n}_{\alpha' p'})\overset{o}{n}_{\alpha'' p''}(1+\overset{o}{n}_{\alpha'' p''})\equiv-\overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})K_{\alpha,\alpha'\alpha''}(p,p'p'');$$

$$\{g_{\alpha p},h_{\alpha p}\}\equiv\sum_{\alpha\alpha'}\frac{1}{h^3}\int d^3p d^3p' \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})g_{\alpha p}K_{\alpha\alpha'}(p,p')h_{\alpha' p'},$$

$$\langle g_{\alpha p}\rangle\equiv\sum_{\alpha}\frac{1}{h^3}\int d^3p\, \overset{o}{n}_{\alpha p}(1+\overset{o}{n}_{\alpha p})g_{\alpha p}. \tag{21}$$

Phonon hydrodynamic equations for energy density ε and momentum density π_l have the form

$$\frac{\partial\varepsilon}{\partial t}=-\frac{\partial q_n}{\partial x_n}, \qquad \frac{\partial\pi_l}{\partial t}=-\frac{\partial t_{ln}}{\partial x_n}+R_l. \tag{22}$$

We write down the general expressions for the fluxes and frictional force density

$$t_{ln}=P\delta_{nl}-\eta_{nl,ms}\frac{\partial\pi_m}{\partial x_s}-\alpha_{nl,ms}\frac{\partial T}{\partial x_m}\pi_s+\beta_{nl,ms}\pi_m\pi_s+O(g^2\lambda^0),$$

$$q_n=h\pi_n-\kappa\frac{\partial T}{\partial x_n}+\sum_{m+s=3}O(g^m\lambda^s), \qquad R_l=-\nu\pi_l-\nu_T\frac{\partial T}{\partial x_l}+\sum_{m+s=3}O(g^m\lambda^s) \tag{23}$$

where ν, ν_T are friction coefficients, κ is thermal conductivity, $\eta_{nl,ms}$ is viscosity tensor, h is drift energy transfer coefficient, $\mu_{nl,ms}$ is drift momentum transfer coefficient, $\alpha_{nl,ms}$ is convective momentum transfer coefficient. These quantities are given by formulas

$$h=\frac{1}{3}\langle\varepsilon_{\alpha p}\frac{\partial\varepsilon_{\alpha p}}{\partial p_n}A_{\alpha n}(p)\rangle, \qquad \nu=\frac{1}{3}\{p_n,A_{\alpha n}(p)\}^U, \qquad \nu_T=\frac{1}{3}\{p_n,C_{\alpha n}(p)\}^U,$$

$$\kappa=-\frac{1}{3}\langle\varepsilon_{\alpha p}\frac{\partial\varepsilon_{\alpha p}}{\partial p_n}C_{\alpha n}(p)\rangle, \qquad \eta_{nl,ms}=-\frac{1}{3}\langle\frac{\partial\varepsilon_{\alpha p}}{\partial p_n}p_l D_{\alpha ms}(p)\rangle \tag{24}$$

via solutions of the integral equations (16), (18), and (19) ($\{g_{\alpha p},h_{\alpha p}\}^U$ is defined by formulas (21) with the contribution of umklapp processes). The tensor structure of expressions (23) is defined by the symmetry of considered dielectrics. The material coefficients in the case of the cubic symmetry of the classes O, O_h, T_h have the structure

$$a_n=0, \quad b_{nl}\sim\delta_{nl}, \quad c_{nlm}=0.$$

The structure of the tensors of the fourth rank is rather complicated and is not discussed here.

4. Moderately low temperatures

The problem of constructing of the phonon hydrodynamics is usually discussed for low temperatures $T \ll T_D$ on the basis of estimations of the contribution of normal and umklapp processes to the collision integral

$$I^N_{\alpha p}(\mathrm{f}) \sim \mu^0 , \qquad I^U_{\alpha p}(\mathrm{f}) \sim \mu^1 ; \qquad \mu \equiv e^{-T_D/T} . \tag{25}$$

In fact this estimation is true not only for $T \ll T_D$ but for moderately low temperatures too. For example, $\mu \simeq 0{,}1$ for $T = T_D / 2$. This allows to build a perturbation theory in powers of μ without limiting ourselves to the main contributions and neglecting exponentially small (at $T \ll T_D$) corrections.

In our paper the perturbation theory in small parameter μ is built for considering moderately low temperatures. In this case the solutions of equations (16)-(19) for the functions $A_{\alpha n}(p)$, $B_{\alpha nl}(p)$, $C_{\alpha n}(p)$, and $D_{\alpha nl}(p)$ and of the corresponding equations for friction coefficients ν, ν_T are sought in the form

$$\begin{aligned} A_{\alpha n} &= A^{[0]}_{\alpha n} + A^{[1]}_{\alpha n} + O(\mu^2) , & \nu &= \nu^{[1]} + \nu^{[2]} + O(\mu^3) ; \\ C_{\alpha n} &= C^{[0]}_{\alpha n} + C^{[1]}_{\alpha n} + O(\mu^2) , & \nu_T &= \nu^{[1]}_T + \nu^{[2]}_T + O(\mu^3) ; \\ B_{\alpha nl} &= B^{[0]}_{\alpha nl} + B^{[1]}_{\alpha nl} + O(\mu^2) , & D_{\alpha nl} &= D^{[0]}_{\alpha nl} + D^{[1]}_{\alpha nl} + O(\mu^2) . \end{aligned} \tag{26}$$

($A^{[s]}$ is a contribution to a quantity A of the order μ^s). Some results of our calculations in the main approximation at moderately low temperature are

$$\begin{aligned} &A^{[0]}_{\alpha n}(p) = \frac{3 p_n}{\langle p^2 \rangle} ; \quad \nu^{[1]} = \frac{\{p_n, p_n\}^U}{\langle p^2 \rangle} , \quad h^{[0]} = \frac{3T^2}{\langle p^2 \rangle} \frac{\partial p}{\partial T} , \quad \nu^{[1]}_T = \frac{1}{3T} \{p_n, C^{[0]}_{\alpha n}(p)\}^U ; \\ &\kappa^{[0]} = \frac{T^2}{3} \{C^{[0]}_{\alpha n}(p), C^{[0]}_{\alpha n}(p)\}^N , \qquad \eta^{[0]}_{nl,ms} = \frac{\langle p^2 \rangle}{3} \{D^{[0]}_{\alpha nl}(p), D^{[0]}_{\alpha ms}(p)\}^N . \end{aligned} \tag{27}$$

The presence of umklapp processes in the system makes steady states possible in it. A steady solution of the hydrodynamic equation for momentum is found in a perturbation theory in gradients of the temperature

$$\pi^{\mathrm{st}}_l = -\frac{1}{\nu} \left(\nu_T + \frac{\partial P}{\partial T} \right) \frac{\partial T}{\partial x_l} + O(g^3) . \tag{28}$$

Energy density in the steady state is

$$q^{\mathrm{st}}_n = -\kappa_{\mathrm{st}} \frac{\partial T}{\partial x_n} + O(g^3) , \qquad \kappa_{\mathrm{st}} = \kappa + \frac{h}{\nu} \left(\nu_T + \frac{\partial P}{\partial T} \right) \tag{29}$$

where the thermal conductivity of the dielectric under consideration in the steady state κ_{st} is introduced. This formula shows that in the absence of umklapp processes, the thermal conductivity $\kappa_{\rm st}=\infty$ (according to (29) in this case $\nu=0$, $\nu_T=0$). The heat conductivity for moderately low temperatures is given by expressions

$$\kappa_{\rm st}^{[-1]}=\frac{3T^2}{\{p_n,p_n\}^U}\left(\frac{\partial P}{\partial T}\right)^2,\qquad \kappa_{\rm st}^{[0]}=\frac{h^{[0]}\nu_T^{[1]}}{\nu^{[1]}} \tag{30}$$

This main contribution $\kappa_{\rm st}^{[-1]}$ coincides with the result obtained by Akhiezer [1, 8, 9] and does not depend on the definition of the drift velocity.

5. Conclusions

The hydrodynamics of the phonon system of dielectrics which is not based on the local equilibrium assumption is constructed for dielectrics of the cubic symmetry classes. The theory is based on a generalization of the Chapman–Enskog method proposed in [6, 7]. A non-hydrodynamic relaxation process in the phonon system of dielectrics is investigated in details in terms of momentum density. This eliminates the problem with the drift velocity definition. The relaxation process is studied at its ending with introducing a new small parameter that can be justified by comparing linear and quadratic relaxation equations [7]. Integral equations of the generalized Chapman–Enskog method are solved approximately under the assumption that contributions of umklapp processes are small compared with those of normal ones that is true for moderately low temperatures. This transforms the Fredholm integral equations of the second kind into equations of the first kind. For the last ones the converged approximate procedure for solving them by the truncated expansion in orthogonal polynomial series can be proposed. Corrections to the Akhiezer result for heat conductivity in a steady state of a dielectric are calculated.

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