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ON HYDRODYNAMICS OF BROWNIAN PARTICLE SYSTEM WITH ACCOUNT FOR RELAXATION PROCESSES

Generalized hydrodynamics for a system of Brownian particles in an equilibrium liquid is constructed with taking into account non-hydrodynamic relaxation of its mass velocity to zero and temperature to the liquid temperature. The investigation is based on the Fokker–Planck kinetic equation. The standard assumption, that states of the system are weakly spatially non-uniform and, therefore, gradients of the hydrodynamic variables are small (small parameter g), is used. The Bogolyubov idea of the functional hypothesis is taken as a basis for the hydrodynamics construction. It is established that one-particle distribution function of the main order in parameter g is given by the Maxwell distribution with parameters depending on coordinates and time. This means that the local equilibrium is present in the system. Equations for functions that determine the distribution function of the first order in the gradients are obtained. On this basis kinetic coefficients of the system (heat conductivity, viscosity and others) are introduced, their anisotropy is connected with the mass velocity of the Brownian particles. Solutions of the equations are considered at the end of the relaxation processes, thus a new small parameter μ is introduced in the theory. The main approximation in this parameter is analyzed and in this approximation the kinetic coefficients are calculated. To this end a spectral theory for the Fokker–Planck operator, that defines collision integral of the system, is elaborated for the three-dimensional case. Eigenfunctions of this operator are expressed through the tensor Hermite polynomials. The proposed approach can be generalized for the higher approximations in the small parameter μ that allows discussing the physical meaning of the parameter μ . The paper generalizes the standard hydrodynamics of the Brownian motion, which assumes that the relaxation processes are over and the system is described only by the diffusion equation.

Keywords: Brownian motion, non-hydrodynamic relaxation processes, generalized hydrodynamics, Hermite tensor polynomials, kinetic coefficients.

1. Introduction

Investigation of hydrodynamic phenomena in the presence of non-hydrodynamic relaxation processes is an important problem of the modern theory of nonequilibrium processes [1]. Such processes can be observed in spatially uniform states of a system. The problem of their consideration consists in the absence of a small parameter that makes investigation on the basis of the Liouville or kinetic equations impossible. In the study of the spatially non-uniform systems gradients of parameters that describe a nonequilibrium state are considered to be small (small parameter g). This is primarily about the distribution function of the system of the main order in the gradients, which is a nonequilibrium one. An assumption about its equilibrium with parametric dependence on coordinates x and time t considered as a basis for constructing a theory of spatially non-uniform states is called the local equilibrium assumption [2]. These problems are discussed in our investigation of the plasma hydrodynamics [3, 4] on the basis of the Landau kinetic equation. It was proposed to consider the relaxation processes at their end, thus introducing a new small parameter in the theory (we denote it by μ). Even in this approach the problem of an approximate solution of Fredholm integral equations of the second kind remains (in particular, the spectral problem for operator of the linearized collision integral). It is done in [3] with the method of a truncated expansion in the Sonine polynomials. However, the convergence of this procedure needs a further investigation. We suppose that an investigation of the mentioned problems in Brownian

motion theory will help to better understanding of the situation. In further we propose to build a perturbation theory in parameter μ on the level of integral equations for functions determining the distribution function of Brownian particles in the first approximation in the gradients. It promises to simplify greatly the theory which was highly complicated in the calculation of contributions to the distribution function of the order $g^1\mu^1$ in the mentioned papers. In this paper a rarefied gas of Brownian particles (B-particles) immersed in equilibrium liquid is investigated. This corresponds to the standard statement of the problem when interaction between Brownian particles is not taken into account.

The paper is organized as it follows. In Section 2 the basic equation of the theory is discussed. In Section 3 a perturbation theory in small parameters g and μ is built and hydrodynamic equations are derived.

2. Basic equation of the theory

Hydrodynamic states of rarefied system of B-particles immersed in equilibrium liquid are investigated on the basis of the Fokker–Planck equation

$$\frac{\partial f_p(x,t)}{\partial t} = -\frac{p_l}{m} \frac{\partial f_p(x,t)}{\partial x_l} + I_p(f(x,t)) \quad (1)$$

with the collision integral

$$I_p(f) = \mathbf{L}f_p, \quad \mathbf{L}f_p \equiv \gamma \frac{\partial}{\partial p_l} \left(\frac{\partial}{\partial p_l} + \frac{p_l}{mT_0} \right) f_p \quad (2)$$

where γ is a positive constant, m is B-particle mass, T_0 is temperature of the liquid (see derivation of this equation, for example, in paper [5]). Energy density $\varepsilon(x,t)$, momentum density $\pi_l(x,t)$ and mass density $\sigma(x,t)$ are defined by formulas

$$\int d^3 p \varepsilon_p f_p(x,t) = \varepsilon(x,t), \quad \int d^3 p p_l f_p(x,t) = \pi_l(x,t), \quad \int d^3 p m f_p(x,t) = \sigma(x,t) \quad (3)$$

($\varepsilon_p \equiv p^2/2m$). Hydrodynamic equations are a consequence of conservation (alteration) laws in the differential form

$$\begin{aligned} \frac{\partial \varepsilon(x,t)}{\partial t} &= -\frac{\partial q_n(x,t)}{\partial x_n} + \frac{3\gamma}{m^2} \sigma(x,t) - \frac{2\gamma}{mT_0} \varepsilon(x,t), \\ \frac{\partial \pi_l(x,t)}{\partial t} &= -\frac{\partial t_n(x,t)}{\partial x_n} - \frac{\gamma}{mT_0} \pi_l(x,t), \quad \frac{\partial \sigma(x,t)}{\partial t} = -\frac{\partial \pi_l(x,t)}{\partial x_l} \end{aligned} \quad (4)$$

where flux densities of energy q_n and momentum t_n are introduced as functionals of the distribution function by the formulas

$$q_n(x,t) = \int d^3 p \varepsilon_p \frac{p_n}{m} f_p(x,t), \quad t_n(x,t) = \int d^3 p p_l \frac{p_n}{m} f_p(x,t). \quad (5)$$

The mass velocity of the B-particle system $v_l(x,t)$ as a hydrodynamic variable is defined by the formula

$$\pi_l(x,t) = \sigma(x,t) v_l(x,t) \quad (6)$$

(see, for example, [6]). In hydrodynamics besides the laboratory reference system (LRS) the accompanying reference system (ARS) is widely used. Transition from LRS to ARS is performed by the formulas

$$\varepsilon = \varepsilon^0 + \sigma v^2 / 2, \quad q_n = q_n^0 + t_{nl}^0 v_l + (\varepsilon^0 + \sigma v^2 / 2), \quad t_{ln} = t_{ln}^0 + \sigma v_l v_n \quad (7)$$

where density of energy, flux densities of energy and momentum in the ARS are given by expressions

$$\begin{aligned} \varepsilon_n^0(x, t) &= \int d^3 p \varepsilon_p f_p^0(x, t), & q_n^0(x, t) &= \int d^3 p \varepsilon_p \frac{p_n}{m} f_p^0(x, t), \\ t_{ln}(x, t) &= \int d^3 p p_l \frac{p_n}{m} f_p^0(x, t). \end{aligned} \quad (8)$$

These expressions include the distribution function in the ARS $f_p^0(x, t)$

$$f_p^0(x, t) \equiv f_{p+m v(x, t)}(x, t) \quad (9)$$

(a quantity A in the ARS is denoted by A^0).

The temperature of the B-particle system is defined by the formula of the ideal gas theory

$$\varepsilon^0 = 3\sigma T / 2m \quad (10)$$

because the system is rarefied one (see, for example, [6]). Hydrodynamic equations with use of the formulas (4), (6), (7), and (10) take the form

$$\begin{aligned} \frac{\partial \sigma}{\partial t} &= -\frac{\partial \sigma v_n}{\partial x_n} \equiv L_0, & \frac{\partial v_l}{\partial t} &= -v_n \frac{\partial v_l}{\partial x_n} - \frac{1}{\sigma} \frac{\partial t_{ln}^0}{\partial x_n} - \lambda v_l \equiv L_l, \\ \frac{\partial T}{\partial t} &= -v_n \frac{\partial T}{\partial x_n} - \frac{2m}{3\sigma} \left(\frac{\partial q_n^0}{\partial x_n} + t_{ln}^0 \frac{\partial v_l}{\partial x_n} \right) - 2\lambda(T - T_0) \equiv L_4 \quad \left(\lambda \equiv \frac{\gamma}{mT_0} \right). \end{aligned} \quad (11)$$

These equations show that relaxation processes of the velocity attenuation and the temperature equalization are present in the system. In spatially uniform states the relaxation is described by the formulas

$$v_l(t) = v_l(0)e^{-\lambda t}, \quad T(t) = T_0 + [T(0) - T_0]e^{-2\lambda t}. \quad (12)$$

In order to obtain the hydrodynamic equations, the right sides of the equations (4) have to be expressed through the hydrodynamic variables $\xi_\mu(x, t)$

$$\xi_0(x, t) \equiv T(x, t), \quad \xi_l(x, t) \equiv v_l(x, t), \quad \xi_4(x, t) \equiv \sigma(x, t). \quad (13)$$

This can be achieved by searching the distribution function $f_p(x, t)$ in the form of a functional $f_p(x, \xi)$ of variables $\xi_\mu(x)$. The fact that the distribution function $f_p(x, t)$ takes this form in the natural evolution (see [6]) is called the functional hypothesis which is written in the form

$$f_p(x, t) \xrightarrow{t \gg \tau_0} f_p(x, \xi(t)). \quad (14)$$

The formula contains a characteristic time τ_0 and has to satisfy the condition $\tau_0 \ll \lambda^{-1}$ because in our consideration the relaxation is continued (see (12)).

In these terms the hydrodynamic equations (11) take the form at $t \gg \tau_0$

$$\frac{\partial \xi_\mu(x, t)}{\partial t} = L_\mu(x, \xi(t), f(\xi(t))). \quad (15)$$

According to the kinetic equation (1), the functional $f_p(x, \xi)$ satisfies the equation

$$\sum_{\mu} \int d^3 x' \frac{\delta f_p(x, \xi)}{\delta \xi_{\mu}(x')} L_{\mu}(x', \xi, f(\xi)) = -\frac{p_l}{m} \frac{\partial f_p(x, \xi)}{\partial x_l} + I_p(f(x, \xi)). \quad (16)$$

One has to add conditions that define the parameters $\xi_{\mu}(x)$ and are a consequence of the definitions (3), (6), and (10) to this equation

$$\begin{aligned} \int d^3 p m f_p(x, \xi) &= \sigma(x), & \int d^3 p p_l f_p(x, \xi) &= \sigma(x) v_l(x), \\ \int d^3 p \epsilon_p f_p(x, \xi) &= \frac{3}{2m} \sigma(x) T(x) + \frac{1}{2} \sigma(x) v(x)^2. \end{aligned} \quad (17)$$

Equation (16) is a rather complicated nonlinear integro-differential equation and can be solved only in a perturbation theory. However, one has to note that kinetic equation (1) is exactly solvable one and it would be interesting to obtain the functional $f_p(x, \xi)$ by another way (see discussion of the B-particle system hydrodynamic evolution after the ending of the relaxation in [6]).

3. Hydrodynamic equations for B-particle system

Solutions of equation (16) for the distribution function $f_p(x, \xi)$, fluxes q_n^0 , i_{ln}^0 , and right sides $L_{\mu}(x, \xi, f(\xi))$ of the hydrodynamic equations (11) are sought together in the perturbation theory in the gradients of the hydrodynamic variables

$$f_p(x, \xi) = f_p^{(0)} + f_p^{(1)} + O(g^2), \quad L_{\mu}(x, \xi, f(\xi)) = L_{\mu}^{(0)} + L_{\mu}^{(1)} + L_{\mu}^{(2)} + O(g^3), \quad (18)$$

with using the estimate

$$\partial^s \xi_{\mu}(x) / \partial x_{n_1} \dots \partial x_{n_s} \sim g^s \quad (g \equiv l_{fp} / L). \quad (19)$$

Here l_{fp} is a free path length of the B-particles, L is a characteristic length at which parameters $\xi_{\mu}(x)$ are substantially changed ($A^{(s)}$ hereafter denotes contribution of the order g^s to a quantity A).

It is easy to see that equation (16) with additional conditions (17) in the main in the gradients approximation has the exact solution coinciding with the Maxwell distribution

$$f_p^{(0)}(x, \xi) = w_p(\xi(x)), \quad w_p(\xi) = w_{p-mv}^0(\xi), \quad w_p^0(\xi) \equiv \frac{n}{(2\pi m T)^{3/2}} e^{-\frac{\epsilon_p}{T}} \quad (20)$$

because the identity

$$-\frac{\partial w_p(\xi)}{\partial T} 2\lambda(T - T_0) - \frac{\partial w_p(\xi)}{\partial v_l} \lambda v_l = I_p(w(\xi)) \quad (21)$$

is true. The obtained result (20) means that in B-particle hydrodynamics the local equilibrium is present. Note, we do not know references to this result in the literature.

On the basis of the formulas (8), (11), and (20) the following expressions for the fluxes in the ARS and for the right sides of the hydrodynamic equations

$$\begin{aligned} q_n^{0(0)} &= 0, & i_{ln}^{0(0)} &= p \delta_{ln} \quad (p \equiv \sigma T / m); \\ L_0^{(1)} &= -v_n \frac{\partial T}{\partial x_n} - \frac{2T}{3} \frac{\partial v_n}{\partial x_n}, & L_l^{(1)} &= -v_n \frac{\partial v_l}{\partial x_n} - \frac{1}{m\sigma} \frac{\partial \sigma T}{\partial x_n} \end{aligned} \quad (22)$$

are obtained (p is pressure of the B-particles).

Taking into account the rotational invariance, the solution of the equation (16) in the first order in gradients can be written in the form

$$\mathbf{f}_p^{(1)} = w_p^o \left[A_l(p) \frac{\partial T}{\partial x_l} + B_{nl}(p) \frac{\partial v_n}{\partial x_n} \right]_{p \rightarrow p-mv} \quad (23)$$

where $A_l(p)$, $B_{nl}(p)$, $C_l(p)$ are some functions of the corresponding tensor dimensionality. Equation (16) gives the following equations for them

$$\begin{aligned} w_p^o \frac{1}{2mT} \left(\frac{p^2}{mT} - 5 \right) p_l &= \mathbf{L} w^o A_l(p) + \lambda \left(2 + m v_l \frac{\partial}{\partial p_l} \right) w_p^o A_l(p), \\ w_p^o \frac{1}{mT} \left(p_n p_l - \frac{1}{3} \delta_{nl} p^2 \right) &= \mathbf{L} w^o B_{nl}(p) + \lambda \left(1 + m v_l \frac{\partial}{\partial p_l} \right) w_p^o B_{nl}(p), \end{aligned} \quad (24)$$

Additional conditions (17) take the form

$$\begin{aligned} \langle A_l(p) \rangle &= 0, \quad \langle p_n A_l(p) \rangle = 0, \quad \langle \varepsilon_p A_l(p) \rangle = 0; \\ \langle B_{nl}(p) \rangle &= 0, \quad \langle p_m B_{nl}(p) \rangle = 0, \quad \langle \varepsilon_p B_{nl}(p) \rangle = 0 \end{aligned} \quad (25)$$

with account for the notation

$$\langle h_p \rangle = \int d^3 p w_p^o h_p \quad (26)$$

(h_p is some function). Distribution function (23) with expressions (8) and (11) gives the following expressions for the energy and momentum fluxes in the ARS and the right sides of the hydrodynamic equations

$$\begin{aligned} q_n^{0(1)} &= -\kappa_{nm} \frac{\partial T}{\partial x_m} - \alpha_{nsm} \frac{\partial v_s}{\partial x_m}, \quad t_{nl}^{0(1)} = -\beta_{nlm} \frac{\partial T}{\partial x_m} - \eta_{nlsm} \frac{\partial v_s}{\partial x_m}, \\ L_l^{(2)} &= -\frac{1}{\sigma} \frac{\partial t_{ln}^0}{\partial x_n}, \quad L_4^{(2)} = -\frac{2m}{3\sigma} \left(\frac{\partial q_n^{0(1)}}{\partial x_n} + t_{ln}^{0(1)} \frac{\partial v_l}{\partial x_n} \right). \end{aligned} \quad (27)$$

Here the coefficients of the heat conductivity κ_{nm} , viscosity η_{nlms} and additional kinetic coefficients α_{nsm} , β_{nlms}

$$\begin{aligned} \kappa_{nm} &= \langle \varepsilon_p p_n A_m(p) \rangle / m, \quad \alpha_{nms} = \langle \varepsilon_p p_n B_{ms}(p) \rangle / m, \\ \beta_{nlm} &= \langle p_n p_l A_m(p) \rangle / m, \quad \eta_{nlsm} = \langle p_n p_l B_{sm}(p) \rangle / m \end{aligned} \quad (28)$$

are introduced. They are tensors because solutions of the equations (24) depend on the mass velocity v_n in the LRS. Equations (24) are rather complicated and, therefore, we limit ourselves to the investigation of hydrodynamics at the end of relaxation processes. At this stage a new small parameter μ is introduced by the estimates

$$v_l \sim \mu, \quad T - T_0 \sim \mu \quad (\mu \ll 1). \quad (29)$$

Functions $A_l(p)$, $B_{nl}(p)$ are sought in the form of expansions in the powers of μ

$$A_l(p) = A_l^{[0]} + A_l^{[1]} + O(\mu^2), \quad B_{nl}(p) = B_{nl}^{[0]} + B_{nl}^{[1]} + O(\mu^2). \quad (30)$$

($A^{[s]}$ denotes a contribution of the order μ^s to a quantity A).

Further we will need spectral properties of the Fokker–Planck operator which defines the collision integral of the integral equation (1). Its right side eigenfunctions $\varphi_{n_1 \dots n_s}(p)$ are expressed through tensor Hermite polynomials $H_{n_1 \dots n_s}(x)$

$$\mathbf{L}\varphi_{n_1 \dots n_s}(p) = -s\lambda\varphi_{n_1 \dots n_s}(p), \quad \varphi_{n_1 \dots n_s}(p) = w_p^0 H_{n_1 \dots n_s}(q) \quad q \equiv p / (mT_0)^{1/2} \quad (31)$$

and satisfy the completeness condition of the form

$$\sum_{0 \leq s < \infty} \frac{1}{s!} \varphi_{n_1 \dots n_s}(p) \varphi_{n_1 \dots n_s}(p') w_p^{0-1} = \delta(p - p'). \quad (32)$$

In the one-dimensional case the Fokker–Planck operator spectrum is investigated in book [7]. The tensor Hermite polynomials were introduced in the paper by Grad [8] and can be defined by the formula

$$H_{n_1 \dots n_s}(x) = (-1)^s e^{\frac{x^2}{2}} \frac{\partial^s}{\partial x_{n_1} \dots \partial x_{n_s}} e^{-\frac{x^2}{2}} = \prod_{1 \leq i \leq s} (x_{n_i} - \frac{\partial}{\partial x_{n_i}}) 1. \quad (33)$$

Their orthogonality condition has the form

$$\int d^3 x H_{l_1 \dots l_s}(x) H_{l'_1 \dots l'_s}(x) (2\pi)^{-3/2} e^{-x^2/2} = \delta_{ss'} \sum_{\sigma} \delta_{l'_1 \sigma(1)} \dots \delta_{l'_s \sigma(s)} \quad (34)$$

where the sum over all permutations of the subscripts $1, \dots, s$ is taken. The first Hermite tensor polynomials are given by the formulas

$$\begin{aligned} H_{n_1 \dots n_s}(x)|_{s=0} = 1, \quad H_l(x) = x_l, \quad H_m(x) = x_n x_l - \delta_{nl}, \\ H_{nlm}(x) = x_n x_l x_m - (x_n \delta_{lm} + x_l \delta_{nm} + x_m \delta_{nl}). \end{aligned} \quad (35)$$

An arbitrary momentum function $u(p)$ can be expanded over the eigenfunctions of the Fokker–Planck operator by expressions

$$u(p) = \sum_{0 \leq s < \infty} c_{n_1 \dots n_s} \varphi_{n_1 \dots n_s}(p), \quad c_{n_1 \dots n_s} = \frac{1}{s!} \int d^3 p u(p) \varphi_{n_1 \dots n_s}(p) w_p^{0-1}. \quad (36)$$

According to (24) and (35), in the main approximation of the perturbation theory in the small parameter μ it is necessary to solve the following equations

$$\begin{aligned} \mathbf{L}w_p^0 A_l^{[0]}(p) + 2\lambda w_p^0 A_l^{[0]}(p) &= \frac{1}{2(mT_0)^{1/2}} w_p^0 H_{lmn}(q), \\ \mathbf{L}w_p^0 B_{nl}^{[0]}(p) + \lambda w_p^0 B_{nl}^{[0]}(p) &= w_p^0 [H_{nl}(q) - \frac{1}{3} \delta_{nl} H_{mmm}(q)]. \end{aligned} \quad (37)$$

Taking into account (31), from here we obtain the functions $A_l^{[0]}(p)$, $B_m^{[0]}(p)$

$$\begin{aligned} A_l^{[0]}(p) &= -\frac{(mT_0)^{1/2}}{2\gamma} H_{lmn}(q) = -\frac{1}{2\gamma} p_l \left(\frac{p^2}{mT_0} - 5 \right), \\ B_{nl}^{[0]}(p) &= -\frac{mT_0}{\gamma} [H_{nl}(q) - \frac{1}{3} \delta_{nl} H_{mmm}(q)] = -\frac{1}{\gamma} (p_n p_l - \frac{1}{3} p^2 \delta_{nl}) \end{aligned} \quad (38)$$

satisfying the additional conditions (25). Now on the basis of (28) in the main approximation in μ we get expressions for kinetic coefficients of the B-particle system

$$\kappa_{nm}^{[0]} = \kappa \delta_{nm}, \quad \kappa \equiv \frac{5}{2} \frac{\sigma T_0^2}{m\gamma}; \quad \alpha_{nsm} = 0,$$

$$\eta_{lnsm}^{[0]} = \eta(\delta_{ls}\delta_{nm} + \delta_{lm}\delta_{ns} - \frac{2}{3}\delta_{ln}\delta_{sm}), \quad \eta \equiv \frac{3}{2} \frac{\sigma T_0^2}{\gamma}; \quad \beta_{nlm} = 0. \quad (39)$$

Note that one can calculate the kinetic coefficients in the next approximations in the small parameter μ in the same way. It is also interesting to investigate modes of the obtained hydrodynamic equations. Both problems will be considered in a next paper.

4. Conclusions

Hydrodynamic equations for rarefied system (gas) of the Brownian particles immersed in equilibrium liquid were constructed with account for non-hydrodynamic processes of the mass velocity v_n and temperature T relaxation. Analogously to our papers [3, 4], the relaxation was investigated near its ending and kinetic coefficients were calculated in the main approximation in the corresponding small parameter μ . However, a fruitful idea to build a perturbation theory in powers of μ not for the B-particle distribution function but for the functions $A_i(p)$ and $B_{nl}(p)$ that determine the contribution to f_p of the first order in gradients $f_p^{(1)}$ was proposed. This confirms our research experience in the investigation of plasma in papers [3, 4]. Therefore, it is planned to return to the problem discussed in [3] taking into account the mentioned. The statement that the Maxwell distribution with the velocity v_n and temperature T gives an exact expression for the distribution function f_p in the main in gradients approximation is our important result. This shows the possibility of local equilibrium states in the B-particle system as a result of its natural evolution. References for this simple (in our approach) result were not found in the literature by us. In our opinion, it would be important to continue the present paper investigation with using the method elaborated in [9].

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