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## CONSERVATION LAWS BASED ON THE NONLOCAL COLLISION INTEGRAL IN THE CASE OF SMALL INTERACTION

The system under consideration is a weakly non-uniform one-component gas with weak potential interaction. The basis of this paper is the kinetic equation up to the terms of the second order in small potential interaction with the general nonlocal collision integral. This paper is devoted to the particle number, momentum and energy conservation laws. It is shown that the system particle number and momentum are conserved. Also it is shown that although the kinetic energy of the system is conserved on the basis of the local collision integral, it is not conserved on the basis of the nonlocal collision integral. Only the total system energy, which is the sum of the kinetic and potential energies, is conserved on the basis of the nonlocal collision integral. Time evolution equations for the particle number density, momentum density and total energy density and the corresponding fluxes are obtained in terms of the one-particle distribution function in the leading order in small spatial gradients of the one-particle distribution function. In contrast to the standard hydrodynamics based on the Landau kinetic equation, these fluxes contain the terms of the first and second order in small potential interaction.

**Keywords:** weak potential interaction, small gradients, nonlocal collision integral, conservation laws, total energy flux, momentum flux.

### 1. Introduction

In this paper we consider a one-component weakly non-uniform gas with small potential interaction. Usually the hydrodynamics of a system is built on the basis of the local collision integral (see, for example, the investigation of hydrodynamics of a granular system on the basis of the Boltzmann kinetic equation [1] and plasma hydrodynamics on the basis of the Landau kinetic equation [2]). The system distribution function is obtained up to the first order in small gradients, and the kinetic coefficients of the system are obtained. However, the Burnett approximation (i.e. the theory of the second order in small gradients) meets difficulties [3], the results of the Burnett approximation may not refine the results of the first order in small gradients.

Maybe, these difficulties can be overcome on the basis of the nonlocal collision integral. The local collision integral is the collision integral in terms of the one-particle distribution function only in the leading order in gradients. So, some terms can be lost during the investigation of system hydrodynamics in higher-than-the-leading orders in gradients. Thus the problem of hydrodynamics investigation on the basis of the nonlocal collision integral is an important one. This paper is based on the kinetic equation up to the second order in small potential interaction with the general nonlocal second-order collision integral [4, 5]. The particle number, momentum and energy conservation laws are investigated in a perturbation theory in small interaction and in small gradients of the one-particle distribution function. It is shown that although the system kinetic energy is conserved on the basis of the local collision integral, it is not conserved on the basis of the nonlocal collision integral. The total energy, which is the sum of the kinetic and potential energies, is conserved on the basis of the nonlocal collision integral. So the system total energy should be used as a reduced description parameter instead of the system kinetic energy and the temperature definition should be based on the equilibrium expression for the total system energy density.

As known, the expressions for the fluxes of the conserved quantities in terms of the one-particle distribution function are very important for the investigation of system hydrodynamics. In this paper the particle number, momentum and energy flux densities are obtained in the leading order in small gradients.

The obtained results can be the basis of further investigation of system hydrodynamics with the nonlocal collision integral with the help of the Chapman–Enskog method. As is known, the consistent hydrodynamics of the Landau–Vlasov kinetic equation meets difficulties [6]. Maybe, these difficulties can be overcome on the basis of the proposed approach. The proposed approach may also be generalized to the case of dissipative systems in an external random field [5, 7–9].

The paper is organized as follows. In Sec. 2 the conservation laws are investigated, and in Sec. 3 the corresponding fluxes are obtained.

## 2. Particle number, momentum and energy conservation laws

The paper is based on the kinetic equation in the case of small potential interaction up to the second order in small interaction [4]:

$$\begin{aligned} \frac{\partial f(\chi_1, t)}{\partial t} &= -\frac{p_{1n}}{m} \frac{\partial f(\chi_1, t)}{\partial x_{1n}} + \frac{\partial f(\chi_1, t)}{\partial p_{1n}} \frac{\partial}{\partial x_{1n}} \int d\chi_2 V_{12} f(\chi_2) + I(\chi_1, f), \\ \chi &= \{\mathbf{x}, \mathbf{p}\}, \quad V_{12} \equiv V(|\mathbf{x}_1 - \mathbf{x}_2|), \\ I(\chi_1, f) &= \frac{\partial}{\partial p_{1n}} \int d\chi_2 F_{12n} \int_{-\infty}^0 d\tau F_{12\tau} \left( \frac{\partial}{\partial p_{1l}} - \frac{\tau}{m} \frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m} \frac{\partial}{\partial x_{2l}} \right) f(\chi_1) f(\chi_2), \\ F_{12n} &= -\frac{\partial V_{12}}{\partial x_{1n}}, \quad F_{12\tau} = -\frac{\partial}{\partial x_{1l}} V \left( \left| \mathbf{x}_1 - \mathbf{x}_2 + \frac{\tau}{m} (\mathbf{p}_1 - \mathbf{p}_2) \right| \right) \end{aligned} \quad (1)$$

where  $V(|\mathbf{x}|)$  is the pair system potential,  $f(\chi, t)$  is the one-particle distribution function, and  $I(\chi_1, f)$  is the general nonlocal second-order collision integral.

The definitions of the particle, momentum and kinetic energy densities in terms of the one-particle distribution function are standard ones:

$$n(\mathbf{x}, t) = \int d\mathbf{p} f(\chi, t), \quad \pi_n(\mathbf{x}, t) = \int d\mathbf{p} p_n f(\chi, t), \quad \varepsilon_{\text{kin}}(\mathbf{x}, t) = \int d\mathbf{p} \frac{p^2}{2m} f(\chi, t). \quad (2)$$

So, the expressions for the system particle number, momentum and kinetic energy are

$$N(t) = \int d\chi f(\chi, t), \quad \mathbf{\Pi}(t) = \int d\chi \mathbf{p} f(\chi, t), \quad E_{\text{kin}}(t) = \int d\chi \frac{p^2}{2m} f(\chi, t). \quad (3)$$

The exact expression for the system potential energy is given in [4]:

$$E_{\text{pot}}(t) = \frac{1}{2} \int d\chi_1 d\chi_2 f_2(\chi_1, \chi_2, f(t)) V_{12} \quad (4)$$

where  $f_2(\chi_1, \chi_2, f)$  is the two-particle distribution function at the kinetic stage of the system evolution.

The time evolution equation for the quantities (2), (3) can be obtained on the basis of the kinetic equation (1). Note that we restrict ourselves to the terms of the second order in small potential interaction because the kinetic equation (1) is obtained up to the second

order. Obviously we have  $\partial_t N(t) = 0$  because the integral of the divergence is equal to zero. For the momentum time evolution equation after integrating by parts we have

$$\begin{aligned} \partial_t \Pi_n &= (\partial_t \Pi_n)^{(1)} + (\partial_t \Pi_n)^{(2)} = -\int d\chi_1 d\chi_2 f(\chi_1) f(\chi_2) \partial V_{12} / \partial x_{1n} - \\ & - \int d\chi_1 d\chi_2 F_{12n} \int_{-\infty}^0 d\tau F_{12\tau} \left( \frac{\partial}{\partial p_{1l}} - \frac{\tau}{m} \frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m} \frac{\partial}{\partial x_{2l}} \right) f(\chi_1) f(\chi_2), \end{aligned} \quad (5)$$

here and in what follows, the superscript in parentheses denotes the order in small potential interaction (the dimensionless small parameter is denoted by  $\lambda$ ).

On the basis of the Fourier transform

$$V_{12} = (2\pi)^{-3} \int d\mathbf{k} e^{i\mathbf{k}(x_1 - x_2)} V(k), \quad k \equiv |\mathbf{k}| \quad (6)$$

we obtain

$$\begin{aligned} (\partial_t \Pi_n)^{(1)} &= -i(2\pi)^{-3} \int d\chi_1 d\chi_2 d\mathbf{k} f(\chi_1) f(\chi_2) k_n e^{i\mathbf{k}(x_1 - x_2)} V(k) = \{1 \leftrightarrow 2\} = \\ &= -i(2\pi)^{-3} \int d\chi_1 d\chi_2 d\mathbf{k} f(\chi_1) f(\chi_2) k_n e^{-i\mathbf{k}(x_1 - x_2)} V(k) = \{\mathbf{k} \rightarrow -\mathbf{k}\} = \\ &= i(2\pi)^{-3} \int d\chi_1 d\chi_2 d\mathbf{k} f(\chi_1) f(\chi_2) k_n e^{i\mathbf{k}(x_1 - x_2)} V(k) = -(\partial_t \Pi_n)^{(1)}, \end{aligned} \quad (7)$$

so  $(\partial_t \Pi_n)^{(1)} = 0$ . For  $(\partial_t \Pi_n)^{(2)}$  we have

$$\begin{aligned} (\partial_t \Pi_n)^{(2)} &= (2\pi)^{-6} \int d\chi_1 d\chi_2 d\mathbf{k} d\mathbf{k}' V(k) V(k') k'_n k_l e^{i(\mathbf{k} + \mathbf{k}')(x_1 - x_2)} \times \\ & \times \int_{-\infty}^0 d\tau e^{i\tau \mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)/m} \left( \frac{\partial}{\partial p_{1l}} - \frac{\tau}{m} \frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m} \frac{\partial}{\partial x_{2l}} \right) f(\chi_1) f(\chi_2) \end{aligned} \quad (8)$$

which after integrating by parts gives

$$\begin{aligned} (\partial_t \Pi_n)^{(2)} &= 2i(2\pi)^{-6} \int d\chi_1 d\chi_2 d\mathbf{k} d\mathbf{k}' V(k) V(k') k'_n k'_l f(\chi_1) f(\chi_2) e^{i(\mathbf{k} + \mathbf{k}')(x_1 - x_2)} \times \\ & \times \int_{-\infty}^0 d\tau e^{i\tau \mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)/m} \frac{\tau}{m}. \end{aligned} \quad (9)$$

With the help of a trick similar to (7) it can be shown that the integral (9) is equal to zero, so the system momentum is conserved on the basis of the general second-order nonlocal collision integral.

Similarly, we can obtain the time evolution equation for the kinetic energy:

$$\begin{aligned} \partial_t E_{\text{kin}} &= (\partial_t E_{\text{kin}})^{(1)} + (\partial_t E_{\text{kin}})^{(2)}, \\ (\partial_t E_{\text{kin}})^{(1)} &= -i(2\pi)^{-3} m^{-1} \int d\chi_1 d\chi_2 p_{1n} f(\chi_1) f(\chi_2) \int d\mathbf{k} V(k) k_n e^{i\mathbf{k}(x_1 - x_2)}, \\ (\partial_t E_{\text{kin}})^{(2)} &= (2\pi)^{-6} m^{-1} \int d\chi_1 d\chi_2 d\mathbf{k} d\mathbf{k}' V(k) V(k') k_l k'_l f(\chi_1) f(\chi_2) e^{i(\mathbf{k} + \mathbf{k}')(x_1 - x_2)} \times \\ & \times \left( i p_{12n} k'_n \int_{-\infty}^0 d\tau e^{i\tau \mathbf{k} \mathbf{p}_{12}/m} \frac{\tau}{m} - \int_{-\infty}^0 d\tau e^{i\tau \mathbf{k} \mathbf{p}_{12}/m} \right), \quad \mathbf{p}_{12} \equiv \mathbf{p}_1 - \mathbf{p}_2. \end{aligned} \quad (10)$$

The system is assumed to be weakly non-uniform on the scale of the radius of particle interaction, so we can expand  $f(\chi_2)$  into a series in  $\mathbf{x}_2 - \mathbf{x}_1$ :

$$f(\chi_2) = f(\mathbf{x}_1, p_2) + \frac{\partial f(\mathbf{x}_1, p_2)}{\partial x_{1n}} x_{21n} + \frac{1}{2} \frac{\partial^2 f(\mathbf{x}_1, p_2)}{\partial x_{1n} \partial x_{1l}} x_{21n} x_{21l} + \dots, \quad \mathbf{x}_{21} \equiv \mathbf{x}_2 - \mathbf{x}_1, \quad (11)$$

the corresponding dimensionless small parameter that describes the smallness of gradients of the one-particle distribution function is denoted by  $g$ . On the basis of (11) and the integrals (24) and (25) from Appendix with the help of the trick (7) and integrating by parts one can obtain that

$$\begin{aligned} (\partial_t E_{\text{kin}})^{(1,0)} &= (\partial_t E_{\text{kin}})^{(2,0)} = 0, \\ (\partial_t E_{\text{kin}})^{(1,1)} &= -V(k=0) \int d\chi_1 d\mathbf{p}_2 \frac{p_{1n}}{m} f(\chi_1) \frac{\partial f(x_1, p_2)}{\partial x_{1n}}, \\ (\partial_t E_{\text{kin}})^{(2,1)} &= (2\pi^2)^{-1} \int d\chi_1 d\mathbf{p}_2 f(\chi_1) \frac{\partial f(x_1, p_2)}{\partial x_{1l}} \frac{p_{12l}}{p_{12}^2} \int_0^\infty dk k^2 V^2(k), \end{aligned} \quad (12)$$

here and in what follows,  $A^{(n,m)}$  denotes the contribution of the order  $\lambda^n g^m$  to a quantity  $A$ . The zeroth order in gradients for  $\partial_t E_{\text{kin}}$  is calculated on the basis of the first term on the right-hand side of (11), i.e. it corresponds to the local collision integral. So the system kinetic energy is conserved on the basis of the local collision integral. It is the well-known result (see, for example, [2]). However,  $(\partial_t E_{\text{kin}})^{(1,1)} \neq 0$  and  $(\partial_t E_{\text{kin}})^{(2,1)} \neq 0$ , so the kinetic energy of the system is not conserved on the basis of the nonlocal collision integral. Thus if we want to investigate the hydrodynamics on the basis of the nonlocal collision integral, we should not use the kinetic energy as a reduced description parameter – we need another reduced description parameter.

By a similar procedure on the basis of the definitions (3), (4) and on the basis of the following expression for the two-particle distribution function [4]

$$\begin{aligned} f_2(\chi_1, \chi_2, f)^{(0)} &= f(\chi_1) f(\chi_2), \\ f_2(\chi_1, \chi_2, f)^{(1)} &= -\int_{-\infty}^0 d\tau F_{12\tau} \left( \frac{\partial}{\partial p_{1l}} - \frac{\tau}{m} \frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m} \frac{\partial}{\partial x_{2l}} \right) f(\chi_1) f(\chi_2) \end{aligned} \quad (13)$$

it can be shown that up to the second order both in small interaction and in small gradients the system total energy is conserved on the basis of the nonlocal collision integral:

$$\partial_t E = 0, \quad E = E_{\text{kin}} + E_{\text{pot}}. \quad (14)$$

Thus the following reduced description parameters for the hydrodynamics investigation should be used: the particle density, the momentum density, and the total energy density. The definitions of the particle density and the momentum density are given by (2); on the basis of (3) and (4) the definition of the total energy density  $\varepsilon$  is

$$\varepsilon(\mathbf{x}_1, t) = \int d\mathbf{p}_1 \frac{p_1^2}{2m} f(\chi_1, t) + \int d\mathbf{p}_1 d\chi_2 f_2(\chi_1, \chi_2, f(t)) V_{12}. \quad (15)$$

In what follows, we need expressions for the corresponding fluxes.

### 3. Particle number, momentum and total energy fluxes

This section is devoted to the obtaining of the particle number, momentum and total energy flux densities. In this paper we restrict ourselves to the leading approximation in small gradients.

The time evolution equations for the particle, momentum and total energy densities can be obtained on the basis of the definitions (2), (15), the kinetic equation (1), and the expressions (13). Here we restrict ourselves to the second order in small interaction. For the particle density we obviously have

$$\partial_t n(\mathbf{x}, t) = -\frac{1}{m} \frac{\partial \pi_n}{\partial x_n}, \quad (16)$$

so the continuity equation is the same both for the local and the nonlocal collision integrals. For the momentum density one can obtain

$$\begin{aligned} \partial_t \pi_n &= (\partial_t \pi_n)^{(0)} + (\partial_t \pi_n)^{(1)} + (\partial_t \pi_n)^{(2)}, \\ (\partial_t \pi_n(\mathbf{x}_1, t))^{(0)} &= -\frac{\partial}{\partial x_{1l}} \int d\mathbf{p}_1 \frac{p_{1l} p_{1n}}{m} f(\chi_1), \\ (\partial_t \pi_n(\mathbf{x}_1, t))^{(1)} &= -i(2\pi)^{-3} \int d\mathbf{p}_1 d\chi_2 d\mathbf{k} f(\chi_2) f(\chi_1) V(k) k_n e^{i\mathbf{k}(\mathbf{x}_1 - \mathbf{x}_2)}, \\ (\partial_t \pi_n(\mathbf{x}_1, t))^{(2)} &= (2\pi)^{-6} \int d\mathbf{p}_1 d\chi_2 d\mathbf{k} d\mathbf{k}' V(k) V(k') k_l k'_n e^{i(\mathbf{k} + \mathbf{k}')(\mathbf{x}_1 - \mathbf{x}_2)} \times \\ &\quad \times \int_{-\infty}^0 d\tau e^{i\tau \mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)/m} \left( \frac{\partial}{\partial p_{1l}} - \frac{\tau}{m} \frac{\partial}{\partial x_{1l}} - \frac{\partial}{\partial p_{2l}} + \frac{\tau}{m} \frac{\partial}{\partial x_{2l}} \right) f(\chi_1) f(\chi_2). \end{aligned} \quad (17)$$

Here  $(\partial_t \pi_n(\mathbf{x}_1, t))^{(0)}$  coincides with the known expression in the framework of standard hydrodynamics based on the Landau kinetic equation with the local collision integral.

With the help of the expansion (11) one can transform other terms in (17). As a result, for  $(\partial_t \pi_n(\mathbf{x}_1, t))^{(1)}$  we obtain

$$(\partial_t \pi_n)^{(1,0)} = (\partial_t \pi_n)^{(1,2)} = 0, \quad (\partial_t \pi_n(\mathbf{x}_1, t))^{(1,1)} = -\frac{1}{2} V(k=0) \frac{\partial n^2}{\partial x_{1n}}. \quad (18)$$

As known, the momentum conservation law in differential form is

$$\partial_t \pi_n(\mathbf{x}_1, t) = -\partial t_{nl} / \partial x_{1l} \quad (19)$$

where  $t_{nl}$  is the momentum flux density. So, in the first order in small interaction we have

$$t_{nl}^{(1)} = V(k=0) n^2 / 2 + O(g^2). \quad (20)$$

As can be seen,  $t_{nl}^{(1)}$  does not contain terms linear in  $g$ . As known, the kinetic coefficients are the proportionality coefficients between the fluxes in the linear order in  $g$  and the gradients. So, the Vlasov term (i.e. the term in the kinetic equation that is of the first order in small interaction) has no effect on the kinetic coefficients related to  $t_{nl}^{(1)}$ . This fact justifies our result because, as known [10], the Vlasov term is not important for the calculation of system kinetic coefficients.

For  $(\partial_t \pi_n(\mathbf{x}_1, t))^{(2)}$  one can obtain the following:

$$\begin{aligned}
(\partial_t \pi_n(\mathbf{x}_1, t))^{(2,1)} = & -\frac{\partial}{\partial x_{1l}} \frac{m}{2\pi^2} \int d\mathbf{p}_1 d\mathbf{p}_2 f(x_1, p_2) f(\chi_1) \left( \frac{2p_{12l} p_{12n} - p_{12}^2 \delta_{nl}}{p_{12}^4} \times \right. \\
& \left. \times \int_0^\infty dk k^3 V(k) \frac{dV(k)}{dk} - \frac{\delta_{nl}}{p_{12}^2} \int_0^\infty dk k^2 V^2(k) \right), \quad (\partial_t \pi_n(\mathbf{x}_1, t))^{(2,0)} = 0.
\end{aligned} \tag{21}$$

So, we have the following expression for  $t_{nl}$ :

$$\begin{aligned}
t_{nl} = & \int d\mathbf{p}_1 \frac{p_{1l} p_{1n}}{m} f(\chi_1) + \frac{1}{2} \delta_{nl} n^2 V(k=0) + \\
& + \frac{m}{2\pi^2} \int d\mathbf{p}_1 d\mathbf{p}_2 f(\chi_1) f(x_1, p_2) \left( \frac{2p_{12l} p_{12n} - p_{12}^2 \delta_{nl}}{p_{12}^4} \int_0^\infty dk k^3 V(k) \frac{dV(k)}{dk} - \right. \\
& \left. - \frac{\delta_{nl}}{p_{12}^2} \int_0^\infty dk k^2 V^2(k) \right) + O(\lambda^1 g^2, \lambda^2 g^1, \lambda^3).
\end{aligned} \tag{22}$$

By a similar procedure, one can obtain the total energy flux density:

$$\begin{aligned}
\partial \varepsilon(\mathbf{x}_1, t) / \partial t = & -\partial q_n / \partial x_{1n}, \\
q_n = & \int d\mathbf{p}_1 \frac{p_{1l}}{2m} \frac{p_{1n}}{m} f(\chi_1) + \frac{1}{m} V(k=0) n \pi_n + \\
& + \frac{1}{4\pi^2} \int d\mathbf{p}_1 d\mathbf{p}_2 f(\chi_1) f(x_1, p_2) \left( 2p_{1n} \frac{2p_{12l} p_{12n} - p_{12}^2 \delta_{nl}}{p_{12}^4} - \frac{p_{12l}}{p_{12}^2} \right) \times \\
& \times \int_0^\infty dk k^3 V(k) \frac{dV(k)}{dk} - \frac{1}{\pi^2} \int d\mathbf{p}_1 d\mathbf{p}_2 f(\chi_1) f(x_1, p_2) \frac{p_{1l}}{p_{12}^2} \int_0^\infty dk k^2 V^2(k) + \\
& + O(\lambda^1 g^2, \lambda^2 g^1, \lambda^3).
\end{aligned} \tag{23}$$

Only the first term on the right-hand side of (23) coincides with the corresponding expression of standard hydrodynamics on the basis of the Landau kinetic equation with the local collision integral, the other terms come from accounting for the Vlasov term and the non-locality of the collision integral. Also note that the Vlasov term does not lead to contributions to (23) linear in  $g$ . So, the Vlasov term has no effect on the system kinetic coefficients related to the energy flux  $q_n$ , which justifies our result according to [10].

The expressions for the fluxes in terms of the one-component distribution function are very important for the investigation of hydrodynamics. Here we restrict ourselves only to the leading-in- $g$  terms in the fluxes. The linear-in- $g$  terms can also be obtained on the basis of the above-described procedure, but this will be made in another paper.

#### 4. Conclusions

This paper is devoted to the investigation of conservation laws and to the calculation of the corresponding fluxes on the basis of the nonlocal collision integral. The system under consideration is a one-component weakly non-uniform gas (small parameter  $g$ ) with small potential interaction (small parameter  $\lambda$ ). Only the pair collisions between the system particles are taken into account.

The kinetic equation for the considered system is known in the literature [4]. It contains the nonlocal collision integral in the considered orders  $\lambda$  and  $\lambda^2$ . All the calculations are made up to the second order in  $\lambda$ .

First of all, the conservation laws in the system are investigated. It is known that the momentum and the particle number of the system are conserved on the basis of the local Landau collision integral. Here it is shown that these quantities are also conserved on the basis of the nonlocal collision integral.

It is also known that the system kinetic energy is conserved on the basis of the local Landau collision integral. But here it is shown that the kinetic energy of the system is not conserved on the basis of the nonlocal collision integral, and the time evolution equation for the system kinetic energy is obtained (10), (12). It is shown that only the total system energy is conserved on the basis of the nonlocal collision integral; it is conserved up to the second order both in the small parameter  $g$  and in the small parameter  $\lambda$ . The total system energy is the sum of the kinetic and the potential energies.

So, we should use the following set of the reduced description parameters in order to build the system hydrodynamics: the particle density, the momentum density, and the total energy density. Expressions for the corresponding flux densities in terms of the one-particle distribution functions are necessary for further development.

It is shown that the continuity equation (16) coincides both for the local and the nonlocal collision integrals. The momentum flux density and the total energy flux density are obtained in terms of the one-particle distribution function (22), (23). The leading-in-interaction terms of these fluxes coincide with the corresponding fluxes of standard hydrodynamics based on the local Landau collision integral. But the other terms are the corrections which come from taking into account the Vlasov term and the non-locality of the collision integral. It should be stressed that the Vlasov term does not give linear-in-gradients terms to the fluxes. This fact justifies our result because, as is known, the Vlasov term has no effect on the system kinetic coefficients.

Here we have restricted ourselves only to the leading-in-gradients terms in the fluxes. Of course, the linear-in-gradients terms for the fluxes are also of great interest because they are connected with the system kinetic coefficients. They can be obtained by the procedure described in this paper.

The fluxes in higher-than-the-leading orders in gradients will be obtained and the system hydrodynamics will be investigated on the basis of the Chapman–Enskog method in another paper. The results of this paper can be a basis for such an investigation.

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### Appendix. Some useful integrals.

The integrals over  $\mathbf{x}_2$  :

$$\int d\mathbf{x}_2 e^{i\mathbf{k}(x_1-x_2)} = (2\pi)^3 \delta(\mathbf{k}), \quad \int d\mathbf{x}_2 e^{i\mathbf{k}(x_1-x_2)} (x_{2n} - x_{1n}) = i(2\pi)^3 \partial\delta(\mathbf{k})/\partial k_n, \quad (24)$$

$$\int d\mathbf{x}_2 e^{i\mathbf{k}(x_1-x_2)} (x_{2n} - x_{1n})(x_{2l} - x_{1l}) = -(2\pi)^3 \partial^2\delta(\mathbf{k})/\partial k_n \partial k_l.$$

The integrals over  $\tau$  :

$$\int_{-\infty}^0 d\tau \int d\mathbf{k} f(k) e^{i\tau\mathbf{k}\mathbf{p}/m} = 2\pi^2 m p^{-1} \int_0^{\infty} dk k f(k),$$

$$\int_{-\infty}^0 d\tau \frac{\tau}{m} \int d\mathbf{k} f(k) k_l e^{i\tau\mathbf{k}\mathbf{p}/m} = 2\pi^2 i m p_l p^{-3} \int_0^{\infty} dk k f(k),$$

$$\int_{-\infty}^0 d\tau \int d\mathbf{k} f(k) k_l e^{i\tau\mathbf{k}\mathbf{p}/m} = -4\pi i m p_l p^{-2} \int_0^{\infty} dk k^2 f(k),$$

$$\int_{-\infty}^0 d\tau \frac{\tau}{m} \int d\mathbf{k} f(k) k_n k_l e^{i\tau\mathbf{k}\mathbf{p}/m} = 4\pi m (2 p_l p_n - \delta_{nl} p^2) p^{-4} \int_0^{\infty} dk k^2 f(k), \quad (25)$$

$$\int_{-\infty}^0 d\tau \int d\mathbf{k} f(k) k_n k_l e^{i\tau\mathbf{k}\mathbf{p}/m} = \pi^2 m (p^2 \delta_{nl} - p_n p_l) p^{-3} \int_0^{\infty} dk k^3 f(k),$$

$$\int_{-\infty}^0 d\tau \frac{\tau}{m} \int d\mathbf{k} f(k) k_n k_l k_q e^{i\tau\mathbf{k}\mathbf{p}/m} =$$

$$= \pi^2 m i (p^2 (p_l \delta_{nq} + p_n \delta_{lq} + \delta_{nl} p_q) - 3 p_q p_n p_l) p^{-5} \int_0^{\infty} dk k^3 f(k),$$

$$k \equiv |\mathbf{k}|, \quad p \equiv |\mathbf{p}|.$$

Received 14.11.2017