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ON THE QUANTIZATION OF CHARGED BLACK HOLES

In order to build a quantum model of a charged black hole (BH), we introduce a modified description of the spherically symmetric configuration of electromagnetic and gravitational fields. The action of the system in a spherically symmetric space-time decays into two terms for the R- and T-regions respectively. After eliminating a non-dynamic variable, initial actions in these regions take the form of actions for a geodesic in three-dimensional configuration spaces. These configuration spaces are shown to be flat, that allows us to introduce variables, in which the minisuperspace metrics take the pseudo-Euclidean form. According to the method of D.M. Gitman and I.V. Tyutin, we construct the canonical formalism of a dynamical system in the extended phase space in these variables. It turns out that the physical part of the Hamiltonian of the system equals to zero and its wave function obeys only eigenvalue equations for the observed physical quantities: mass and charge. Solving these equations leads to continuous spectra of charge and mass in the considered model of charged BH.

Keywords: black holes, mass function, charge function, Hamiltonian constraint, quantization, mass and charge operators.

1. Introduction

Spherically symmetric models are the simplest configurations for testing the main consequences of quantum gravity and studying the problems that arise when the theory is examined more fully. The general geometrodynamic approach to a spherically symmetric gravitational field was developed in [1], the case of the configuration of electromagnetic and gravitational fields was considered in [2].

The proposed model is based on the observation that the classical spherically symmetric configurations of electromagnetic and gravitational fields are stationary from the point of view of an external observer and they have certain regions of space-time with dynamic behavior. This means that in these regions there is an evolution of the geometry of space-time in time, which is responsible for the quantum-mechanical properties of the black hole model in the problem under consideration. Such models are considered in [3, 4]. In this paper the quantum model of a charged black hole is constructed with using the D.M. Gitman and I.V. Tyutin method [5].

2. Classic description of charged BH

The action for the gravitational and electromagnetic fields in space-time V^4 has the form:

$$S_{tot} = -\frac{1}{16\pi}\int_{V^4}\left\{\frac{c^3}{\kappa}R^{(4)} + \frac{1}{c}F_{\mu\nu}F^{\mu\nu}\right\}\sqrt{-g}\,d^4x \tag{2.1}$$

For a spherically symmetric configuration the electromagnetic field tensor and the interval have the form:

$$F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} \rightarrow F_{ab} = A_{a,b} - A_{b,a} \tag{2.2a}$$

$$ds^2 = h\left(x^0,r\right)\left(dx^0\right)^2 - g\left(x^0,r\right)dr^2 - R^2\left(x^0,r\right)d\sigma^2\,, \tag{2.2b}$$

where $d\sigma^2 = d\theta^2 + \sin^2\theta d\alpha^2$ is angular, a, b = 0, 1. After integrating over the angles and discarding the surface term, the action (2.1) can be reduced to the form

$$S = \int_{V^2}\left(\frac{1}{2c}\frac{\left(A_{1,0} - A_{0,1}\right)^2 R^2}{\sqrt{gh}} + \frac{c^3}{2\kappa}\sqrt{gh}\left[1 + \frac{RR_{,1}}{g}\left(\ln Rh\right)_{,1} - \frac{RR_{,0}}{h}\left(\ln Rg\right)_{,0}\right]\right)d^2x \quad (2.3)$$

Here $X_{,0} \equiv \partial X/\partial x^0$, $X_{,1} \equiv \partial X/\partial x^1$. The surface $R\left(x^0, r\right) = R_g = const$, for which $(\nabla R)^2 = g^{ab}R_{,a}R_{,b} = 0$, divides into one T- and two R-regions. Herewith, $(\nabla R)^2 > 0$ in the T-region, and $(\nabla R)^2 < 0$ in the R-regions. We choose the coordinates in which the metric depends on the spatial coordinate *r* in the R-regions and on the time coordinate x^0 in the T-domain:

$$ds_R{}^2 = h(r)\left(dx^0\right)^2 - g(r)dr^2 - R(r)d\sigma^2 \quad (2.4)$$

$$ds_T{}^2 = h\left(x^0\right)\left(dx^0\right)^2 - g\left(x^0\right)dr^2 - R\left(x^0\right)d\sigma^2 \quad (2.5)$$

The corresponding Lagrangians are equal to

$$L_R = \frac{1}{2c}\frac{A_{0,1}{}^2 R^2}{\sqrt{gh}} + \frac{c^3}{2\kappa}\sqrt{gh}\left[1 + \frac{RR_{,1}}{g}\left(\ln Rh\right)_{,1}\right], \quad (2.6)$$

$$L_T = \frac{1}{2c}\frac{A_{1,0}{}^2 R^2}{\sqrt{gh}} + \frac{c^3}{2\kappa}\sqrt{gh}\left[1 - \frac{RR_{,0}}{h}\left(\ln Rg\right)_{,0}\right]. \quad (2.7)$$

In the R- and T-regions, it is convenient to go over to the new variables:

$$f_R = A_0, u_R = \frac{1}{2}R\left(1+h\right), \quad n_R = \frac{1}{2}R\left(1-h\right), \quad N_R = \sqrt{gh}, \quad (2.8a)$$

$$f_T = -A_1, u_T = \frac{1}{2}R\left(1-g\right), \quad n_T = \frac{1}{2}R\left(1+g\right), \quad N_T = \sqrt{gh}. \quad (2.8.b)$$

Then the Lagrangians (2.6) and (2.7) take the uniform form, which naturally describes a degenerate systems:

$$L_\alpha = \frac{1}{2c}\frac{f_{,\alpha}{}^2\left(u+n\right)^2}{N} + \frac{s}{2}N\left[1 + \frac{u_{,\alpha}{}^2 - n_{,\alpha}{}^2}{N^2}\right] \quad (2.9)$$

Here $s = c^3/\kappa$, α are evolutionary parameters. The Lagrange-Euler equation for N leads to the constraint $\partial L/\partial N = 0$. Expressing N from here and substituting in (2.9), we get:

$$S_{\sup} = s\int\sqrt{du^2 - dn^2 + (u+n)^2 df^2}. \quad (2.10)$$

The equations following from the variational principle $\delta S_{\sup} = 0$ together with the constraint are equivalent to the Einstein equations for the considered configuration. On the other hand, $S_{\sup}$ is an action for the geodesic in the configuration space. Therefore, the action, without constraints, is equivalent to the relativistic particle action in 3D pseudo-Riemannian space with a metric

$$d\Omega^2 = -dn^2 + du^2 + (u+n)^2 df^2 . \tag{2.11a}$$

The metric can be reduced to the pseudo-Euclidean type

$$d\Omega^2 = -da^2 + db^2 + dw^2 \tag{2.11b}$$

by the following conversion of field variables:

$$u = \frac{w^2}{2(a-b)} - b, \quad n = a - \frac{w^2}{2(a-b)}, \quad f = \sqrt{sc}\,\frac{w}{a-b}. \tag{2.12}$$

We substitute the variables (2.12) into the expression (2.9). To construct the canonical formalism, we use the method of D.M. Gitman and I.V. Tyutin [4]. We define the Hamiltonian function in the extended phase space of the system [5]:

$$H = P_i V^i - L^V . \tag{2.13}$$

Here $V^i = \dot{q}^i$ are generalized velocities of the generalized coordinates $q^i = \{a, b, w, N\}$. From $P_i = \partial L / \partial V^i$ we obtain the velocities V_a, V_b, V_w, and primary constraint $P_N = 0$. Substituting the velocities in (2.13) and calculating the Poisson bracket of the primary constraint with the Hamilton function, we find the secondary constraint:

$$\frac{1}{2s}\left[-P_a^2 + P_b^2 + P_w^2 - s^2\right] = 0 \tag{2.14}$$

The Poisson brackets of the constraint (2.14) and the Hamiltonian functions are equal to zero identically, so there are no new constraints. The Poisson brackets of the found constraints are also equal to zero, so they are constraints of the second kind. Their presence means that the system contains nonphysical degrees of freedom. For their explicit separation, we can perform the canonical transformation to new canonical variables $\left(q^i, p_i\right)$ with the generating function of the second type [5]:

$$G_2\left(\alpha, Q^i, p_i\right) = Np_4 + \frac{1}{2p_2}\left[\left(a+b\right)p_2^2 + \left(b-a\right)\left(sp_3 + s^2\right) + \frac{\left(wp_2 - p_1\right)^2}{b-a}\right]. \tag{2.15}$$

In the new variables, the Hamiltonian function and the constraints take the form

$$H = p_4 V_N - \frac{q^4 p_3}{2}; \qquad p_3 = p_4 = 0. \tag{2.16}$$

3. Quantum description of a charged black hole

By virtue of (2.16), the physical part of the Hamiltonian function is identically equal to zero. Thus, in quantization, the wave function of the system is determined only by the eigenvalue equations for the operators of physical quantities: mass and charge. In the classical case they are given by the relations

$$Z = \frac{R^2}{\sqrt{gh}}\left(A_{0,1} - A_{1,0}\right); \qquad M = \frac{s}{2c}R\left(1 + \frac{R_{,0}{}^2}{h} - \frac{R_{,1}{}^2}{g}\right) + \frac{Z^2}{2c^2 R}. \tag{3.1}$$

Performing the transition to variables $\left(q^i, p_i\right)$, we get

$$Z_{ph} = \sqrt{\frac{c}{s}}p_1; \quad M_{ph} = \frac{q^2 p_2{}^2}{sc}. \tag{3.2}$$

Here the question arises of ordering the operators, which we define as follows:

$$\hat{Z} = -i\hbar\sqrt{\frac{c}{s}}\frac{\partial}{\partial q^1}, \qquad \hat{M} = \frac{1}{sc}\frac{\partial}{\partial q^2}q^2\frac{\partial}{\partial q^2} \tag{3.3}$$

The solution of eigenvalue equations for the operators (3.3) leads to the wave function of the spherically symmetric configuration of the gravitational and electromagnetic fields:

$$\Psi(q^1, q^2) = CJ_0\left(\frac{2}{\hbar}\sqrt{scmq^2}\right)\exp\left(\frac{i}{\hbar}\sqrt{\frac{s}{c}}zq^1\right) \tag{3.4}$$

It follows that the mass and charge spectra of a charged black hole are continuous in considered quantum model. This result is compatible with the works [1,2,4,6]. Also, if we set charge $z = 0$, we obtain the wave function for the case of a non-charged black hole. The corresponding expression completely coincides with the result of work [4], which indicates the correctness of the considered method.

4. Conclusions

Thus, the application of the approach of D.M. Gitman and I.V. Tyutin to the considered model leads to continuous mass and charge spectra for a charged black hole. This is consistent with geometry of minisuperspace of considered configuration of gravitational and electromagnetic fields, which appears to be flat. Obtained result agrees with the results of a number of previous works [1-6] and at the same time corresponds the result [4] of considering a non-charged black hole.

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