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CAN THE SPATIAL CURVATURE BE A REASON FOR THE UNIVERSE ACCELERATED EXPANSION?

In this work the possibility for explaining the accelerated expansion of the Universe due to spatial curvature is considered. For this purpose, the inhomogeneous cosmological models based on Stefani solution of Einstein equations are investigated. These models describe universe with shear free matter distribution and include Friedmann models as a particular case. We obtain and present the exactly solvable cosmological model, in which the reason for changing the sign of the cosmological acceleration is the spatial curvature.

Keywords: accelerated expansion of the Universe, Stephani solution, inhomogeneous cosmological models.

1. Introduction

The revolution in cosmology started gradually by the series of discoveries that were awarded by the Nobel Prize in physics in 1978 (discovery of the CMB), 2006 (discovery of the black body spectrum of the CMB anisotropy) and 2011 (discovery of the accelerated expansion of the Universe). These discoveries caused the cardinal review of the modern cosmological picture which is still in its progress.

In this work we focus on the fact that, according to current cosmological observations, our Universe is expanding with acceleration [1]. Two groups of solutions are usually considered that can be used to explain this phenomenon [2].

The first of these is based on the assumption that there exists some (exotic) matter with negative pressure. However, the existence of such matter raises a number of fundamental questions, such as the existence of negative entropy, violation of some energy conditions, a singularity in the future, etc.

On the other hand, a number of authors have generalized the general theory of relativity (GTR) and created new theories of gravitation in which the standard Einstein-Hilbert action is replaced by an arbitrary function of the Ricci scalar R. In such a theory it is possible to describe accelerated expansion of the Universe without introducing dark energy. In recent years, a third approach has been developed which remains within the framework of GTR and does not introduce exotic types of matter, but obtains accelerated expansion of the Universe by considering cosmological models with variable spatial curvature [3-6].

2. Stephani cosmological models

Stephani cosmological models [7] are a particular case of spherically symmetric solutions of the Einstein equations describing a shear free fluid. The energy density in this case depends only on time $\varepsilon = \varepsilon(t)$, the pressure depends on both time and spatial coordinate $p = p(x,t)$. The metric for Stephani universe can be written in the form:

$$ds^2 = \frac{\dot{r}^2(x,t)a^2(t)}{r^2(x,t)\dot{a}^2(t)}dt^2 - r^2(x,t)(dx^2 + d\sigma^2), \qquad (1)$$

where $r^2(x,t) = 2a(t)e^x \big/ \left(1+\zeta(t)a^2(t)e^x\right)$, $\zeta(t)$ is the spatial curvature [6], the dot denotes derivative with respect to time. The equation linking the spatial curvature and the energy density reads

$$\frac{\dot{a}^2}{a^2} + \zeta(t) = \varepsilon(t), \tag{2}$$

and the pressure is given by the expression

$$p(x,t) = -\varepsilon(t) - \dot{\varepsilon}^2 r \big/ 3\dot{r}, \tag{3}$$

Here and below the velocity of light $c = 1$, and the factor $8\pi\gamma/c^4$ is included in the expressions for $\varepsilon(t)$ and $p(x,t)$ so that they are measured in units of cm^{-2}. By a suitable choice of the coordinates it is possible to reduce the spatial part of metric (1) to one of the following forms:

$$dl^2 = \frac{a^2(t)(\chi^2 + \sin^2\chi d\sigma^2)}{\cos^2\dfrac{\chi}{2} + \zeta(t)a^2(t)\sin^2\dfrac{\chi}{2}} = \frac{a^2(t)(d\chi^2 + \sinh^2\chi d\sigma^2)}{\cosh^2\dfrac{\chi}{2} + \zeta(t)a^2(t)\sinh^2\dfrac{\chi}{2}} = \frac{a^2(t)(dR^2 + R^2 d\sigma^2)}{1 + \zeta(t)a^2(t)\dfrac{R^2}{4}} \tag{4}$$

where for each case we have made the substitution of variables $e^x = \tan\chi/2$, $e^x = \tanh\chi/2$, $e^x = R/2$, respectively. It follows directly from equalities (4) that Friedmann models are a particular case of the models under discussion for $\zeta = \pm 1/a^2$,0 and Stephani cosmological models have positive or negative spatial curvature. Stephani universes with flat space are always Friedmann models. The main equation describing Stephani cosmological models is equation (2). Since the energy density depends only on time, we shall write it in the form

$$\varepsilon(t) = a_0^n \big/ a^{n+2}(t), \tag{5}$$

where a_0 is an arbitrary constant. The strong energy condition bounds the interval of possible values of n: $-2 \le n \le 4$. The spatial curvature $\zeta(t)$ can also be expressed in terms of $a(t)$:

$$\zeta(t) = \beta a_0^k \big/ a^{k+2}(t). \tag{6}$$

where β is a dimensionless constant. Thus, in place of (2) we obtain

$$\frac{\dot{a}^2}{a^2} + \beta\frac{a_0^k}{a^{k+2}} = \frac{a_0^n}{a^{n+2}}. \tag{7}$$

Note that for $k = 0$ equation (7) reduces to the Friedmann equation.

Following Ellis [9], we define the deceleration parameter in the following way:

$$q \equiv -l\left(\frac{dl}{d\tau}\right)^{-2}\frac{d^2 l}{d\tau^2}, \tag{8}$$

where τ is proper time, measured along the world line of the particle, and l is length along the world line. From (8) for metric (1) we obtain

$$q = -\frac{1+V}{1+(1+k)V}\left[\frac{\ddot{a}a}{\dot{a}^2} + \frac{kV}{1+V}\right], \tag{9}$$

where $V = \beta(a_0/a)^k R^2/4$, $\beta(a_0/a)^k \tan^2\chi/2$, $\beta(a_0/a)^k \tanh^2\chi/2$, depending on the choice of spatial coordinates in (4). An examination of the expression for the deceleration parameter (9) reveals that to find solutions describing Stephani cosmological models with accelerated expansion it is sufficient to consider the equation (7).

The class of exact solutions describing Stephani cosmological models can be obtained if we assume that $a(t) = a_{Friedmann}(t)$, but further analysis of the expressions for the deceleration parameter for these solutions reveals that the sign of acceleration changes only for models in which the energy density includes the quintessence (or an account of the cosmological constant).

3. A universe in which the sign of acceleration changes only due to curvature

Here we will give an example of an exact solution describing a Stephani universe in which the acceleration changes sign due to curvature. let's rewrite (7) in the form:

$$\frac{\dot{a}^2}{a^2} + \beta\frac{1}{a^{3/2}\sqrt{a_0}} = \frac{a_0}{a^3}. \tag{10}$$

where a_0 is a constant in units of length and β is a dimensionless constant. Equation (10) has an exact solution, which can be written as

$$a = a_0\left[\frac{3}{2}\frac{t}{a_0}\left(1-\beta\frac{3}{8}\frac{t}{a_0}\right)\right]^{2/3}. \tag{11}$$

For $\beta > 0$, i.e., for a closed world, the behavior of $a(t)$ is similar to its Friedmann form. The world is born at $t = 0$ and collapses at $t = a_0 8/3\beta$. For $\beta < 0$ solution (11) describes an open world (with a Lobachevski space), for which the behavior of $a(t)$ differs markedly from its Friedmann version. Calculation of $\ddot{a}$ for solution (11) shows that the acceleration changes sign at

$$a_s = a_0(2/\beta)^{2/3}. \tag{12}$$

We find that for $0 \le a < a_s$ the expansion of the Universe decelerates as in the Friedmann models, but for $a_s \le a < \infty$ the rate of expansion increases. Thus, Stephani cosmological models can describe universes in which the acceleration changes its sign only due to curvature, but not because of the presence of a cosmological constant or quintessence. From solution (11) taking into account (12), we obtain the time at which the universe changes over from decelerated expansion to accelerated one:

$$t_s = 4a_0\left(\sqrt{3}-1\right)\big/3\beta. \tag{13}$$

As was shown in [9], the Hubble constant in Stephani universes has the same form as in Friedmann universes: $H = \dot{a}/a$ Substituting here the solution (11) we obtain

$$H = \frac{2}{3t}\left[1+\frac{3\beta}{4a_0}t\right]\cdot\left[1+\frac{3\beta}{8a_0}t\right]^{-1}. \quad (14)$$

From (14) we see that the ratio between the Hubble constant and time in Stephani universe is almost the same as in Friedmann case. The value of the Hubble constant at the time of the acceleration sign change (13) will be

$$H_s = \sqrt{3}\beta/2a_0 . \quad (15)$$

4. Conclusions

In this work we have considered Stephani cosmological models which describe spacetime for a shear free matter distribution and include Friedmann models as a particular case.

We have obtained an exact solution (11) describing a universe with shear free matter distribution in which the acceleration changes sign only due to curvature.

We have calculated the time of transition from decelerated to accelerated expansion (13).

We have also showed that the connection between time and the Hubble constant (14) is in fact the same as in Friedmann universes.

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