S. F. Lyagushyn

Oles Honchar Dnipropetrovsk National University
e-mail: lyagush.new@gmail.com

DICKE MODEL AND ELECTROMAGNETIC FIELD PROPERTIES

The paper summarizes the long-term Dicke model investigations with a view of revealing the way of correlation development in a system consisting of a large number of two-level quantum emitters interacting via electromagnetic field. As early as in 1953 Dicke predicted the process of self-organizing in such a system, thus generating a short coherent light pulse. After laser invention, the Dicke model was regarded as a way to the analysis of laser systems. Its equilibrium properties and phase transition in such system were under discussion. At the beginning of the 80-ies a consistent nonequilibrium theory for the superradiance was developed in parallel with wide experimental study of the phenomenon. Nevertheless the field state was considered as equilibrium at a certain temperature. Investigations of the problem in Oles Honchar Dnipropetrovsk National University was based on the Bogolyubov reduced description method allowing to include the field properties into the process picture. The quantum-statistical approach implies an approximate depiction of the field with simultaneous correlation functions of different orders. Proceeding from the principles of macroscopic electrodynamics, we propose computer simulation of the superradiance process with using spatial lattice sites.

Keywords: Dicke model, two-level emitters, laser systems, superradiance, nonequilibrium statistical operator, equilibrium medium, correlation functions, computer simulation.

1. Introduction

A system consisting of a great number of two-level quantum emitters interacting via electromagnetic field is described with the Dicke Hamiltonian. Proceeding from the quantum selection rule, in 1953 Dicke predicted the process of self-organizing in such a system, thus generating a short coherent light pulse that was called superradiance. After laser invention, the Dicke model was regarded as a way to the analysis of laser systems though the spontaneous emission determined the nature of processes in it. Then its equilibrium properties and phase transition in such system were under discussion. Rapid growth of nonlinear optics motivated the interest to the Dicke model. The first experimental implementation of the Dicke process dates back to 1973. At the beginning of the 80-ies the principally nonequilibrium nature of the Dicke phenomenon of cooperative spontaneous emission became clear. Some new methods of studying processes in systems of "matter+field" type developed in the Bogolyubov scientific school were applied to this model and a consistent theory for the superradiance was developed in parallel with wide experimental investigation of the phenomenon for the military purpose. Notice that the field state was considered as equilibrium at a certain temperature when the method of eliminating the boson variables was used, though the essence of the Dicke phenomenon is a macroscopic filling of resonant modes of the electromagnetic field. The delay time of the light pulse was identified by the moment of fast changes in the emitter subsystem and the question of the radiation field state was not raised. The modern progress in field state analysis stimulates the interest to more full description of the emission generated by the Dicke system and advances in computational experiments give the tool for such research work. The paper summarizes the approaches used in superradiance theory and puts forward the idea of investigating the field correlations by numerical methods on the basis of the reduced description method that is applied in nonequilibrium quantum statistical physics by scientists of Dnipropetrovsk National University.

© S. F Lyagushyn, 2016

2. Early studies of the Dicke model. An equilibrium approach

During the analysis of a spontaneous emission process in a complex of identical two-level emitters in the foundational paper [1] R. H. Dicke pointed out the following consequence of applying the quantum symmetry requirements to the process: the system should pass such states that could be considered as states of the joint electric dipole moment. This approach to the system of *N* emitters resulted in the anticipation of the radiation with intensity proportional to N^2 after a period of time when the correlations between the emitters arise. No direct interaction of emitters was necessary here; they interact via the electromagnetic field, so the Dicke Hamiltonian had a simple structure

$$\hat{H}_D = \hat{H}_m + \hat{H}_f + \hat{H}_{mf} \tag{1}$$

where $\hat{H}_m$ and $\hat{H}_f$ are Hamilton operators of the systems of emitters (matter) and field, correspondingly, wherein $\hat{H}_{mf}$ describes the interaction between the matter and field [2].

Obviously, two-level emitters can be described by quasispin operators and their direct contribution to the Hamiltonian H_D is

$$H_m = \sum_i \hbar\omega r_i^z \tag{2}$$

where $r_i^z = \frac{1}{2}\sigma_i^z$, σ_i^z is a Pauli matrix, index *i* numerates emitters, the frequency of the working transition ω is accepted to be equal for all the emitters. The free field Hamiltonian is put down as

$$H_f = \sum_k \hbar\omega_k a_k^+ a_k \tag{3}$$

2where index *k* numerates both the wave vector of a mode and its polarization, ω_k is a field mode frequency, and a_k^+ (a_k) are standard operators of creation and annihilation of photons [3]. As usually, we omit the constant terms in H_m and H_f. The Dicke Hamiltonian uses the simplest operator form for H_{mf} with evident physical interpretation:

$$H_{mf} = \sum_{i,k} g_{ik} (r_i^- a_k^+ e^{-i\mathrm{k}\cdot\mathrm{x}_i} + r_i^+ a_k e^{i\mathrm{k}\cdot\mathrm{x}_i}) \,. \tag{4}$$

Here raising and lowering Pauli operators for the *i*-th emitter $r_i^\pm = \frac{1}{2}\left(\sigma_i^x \pm i\sigma_i^y\right)$ are used and vectors $\mathbf{x}_i$ indicate the locations of corresponding point particles. g_{ik} denotes the known coupling parameter of the *i*-th emitter and *k*-th mode [2, 3]. Such expression for H_{mf} can be obtained both from considering the interaction in terms of vector potential of the wave field (Coulomb gauge) and from using the dipole approximation [4]. We can argue that the operator form (4) is gauge invariant and valid for calculations of averages including those of statistical systems if the proper transformations of physical value and statistical operators are applied [5]. There were wide discussions in literature about the correct form of the Dicke system Hamiltonian. We avoid the detailed retelling of them concentrating ourselves on the principal stages of research into the problems of such system behavior. Really the matter-field interaction has a resonant character. Thus, quickly oscillating terms are excluded from (4) and the Hamiltonian (1) with summands explained above is referred to as Dicke Hamiltonian in the rotating wave approximation. Some important features of the Dicke model behavior can be investigated with using the one-mode concentrated system: its dimensions are much less than the wavelength of the generated emission. In this case the rotational symmetry compels us to use the idea of quasiaverages [6]. Emitter frequency spread can be taken into account through some distribution function. A lot of generalizations was proposed and analyzed for the Dicke Hamiltonian [2, 3]. The first years of Dicke model studies coincided with the beginning of the era of lasers. In spite of the well-known stimulated emission role in the laser principle, it was assumed that the Dicke model can be useful for explaining the physical phenomena in lasers. Its attractiveness was based on the existence of an exact solution for the equilibrium properties of such a system in the formal mathematical approach and something in common between these properties and laser generation threshold. Hepp and Lieb [7] computed thermodynamic functions for Dicke model in the limit $N \to \infty$ and shown that the system exhibits a second order phase transition from normal to superradiance. The model was studied thoroughly on the basis of the method of approximating Hamiltonians developed by N. N. Bogolyubov (Jr.) [8]. For the one-mode version of the Dicke Hamiltonian the macroscopic filling of the resonant boson mode $\left\langle a^{+}a \right\rangle \sim N$, similar to the analogous filling in the theory of superfluidity, was substantiated at temperatures less than a certain critical value. But it was clear that the process physics in superradiance is another, that ordering occurs on the way to equilibrium, and we deal with some kind of a "dynamical phase transition".

3. Experimental interest to superradiance. Kinetic equations for a Dicke system in the scheme of boson variable elimination

The analysis of conditions for superradiance observation opened a way to putting it into life. It was done by Feld et al. in experiments with HF molecules in 1973 [9]. Pay attention that in this case we have an opposite relation between the theory and experiment in comparison with such collective quantum phenomena as superfluidity and superconductivity. The experimental achievement [9] fallen behind the Dicke's prediction by 20 years. Superradiance implementation required providing the proper relationship between a number of characteristic times in the system under consideration. Further years showed great interest to the phenomenon in different media because just collective spontaneous emission makes possible a coherent generation in the range of frequencies where mirrors are absent. Hypothetical coherent generators in X-ray and gamma radiation ranges can not be based on the stimulated emission. The creation of such generators may be very prospective for fundamental physics and for practical applications, particularly in the military sphere. The last circumstance strongly influenced the experimental investigation of Dicke's superradiance. Adequate theoretical research was necessary for the development of the scientific direction. Different nonequilibrium statistical methods were applied to the Dicke model and the method of boson variable elimination proved to be the most fruitful at that stage. It was based on the Bogolyubov lemma [10] allowing to express the quantum-statistical averages of the product of a boson operator and arbitrary operator of the "matter+field" system via the averages of their commutator. In such a way the kinetic problem for the joint system can be reduced to the dynamics of the "small" subsystem of emitters where the relaxation processes are slower than those of the field subsystem playing the role of thermostat. The paper of 1981 by Bogolyubov (Jr.), Fam Le

Kien, and Shumovsky [11] presented the consistent hierarchy of kinetic equations for quasispin operators of the Dicke model. In fact, it was a consequence of the evolution equation for an arbitrary operator of the emitter subsystem in Heisenberg representation with substituted formal solutions of the evolution equations for photon operators. The chain was broken and the set of equations was solved with using the collective atomic operators of the emitter subsystem taking into account its geometry

$$R_{\nu}^{\pm} = \sum_i r_i^{\pm} e^{\pm i\nu\cdot x_i} , \qquad R_{\nu}^{z} = \sum_i r_i^{z} e^{i\nu\cdot x_i} . \tag{5}$$

The vector **ν** determines modes in the working volume: $\nu_l = \dfrac{2\pi n_l}{V^{1/3}}$, n_l=0,1,… , l=1, 2, 3.

Time integrals were eliminated due to the assumption of small interaction and Markovian equations were obtained. Proceeding from the evolution equation for an arbitrary atomic operator average, one could obtain the Markovian "Master equation" for the statistical operator of the emitter subsystem.

Many ideas concerning more accurately taking into account various additional factors of the process (external field and pumping influence, losses, competing processes of stimulated emission and many-photon ones) were put forward. In spite of numerous results, we may note somewhat paradoxical situation with the method of boson variable elimination. All conclusions about superradiant generation were made on the basis of evolution picture of emitters due to the Dicke Hamiltonian integral of motion

$$\sum_k a_k^{+} a_k + R_0^z . \tag{6}$$

Quick changes of $\left\langle R_0^z(t) \right\rangle$ mean maximal changes of the total filling of boson modes, i.e. superradiant pulse delay time. The picture of field correlations remains unknown.

4. Dicke model behavior in the framework of the reduced description method. New possibilities and perspectives

No doubt that Dicke superradiance is the most interesting example of system relaxation to equilibrium. New methods of nonequilibrium statistical physics can be tested on this basis. Such efforts were undertaken in relation to the Dicke model by the theorists of Dnipropetrovsk National University since 2000 under the supervision of Professor A. I. Sokolovsky. Our research was based on the Bogolyubov reduced description method developed by the Kharkiv scientific school [12]. Enough detailed presentation of the method is given in the paper [13] in our "Visnyk". The general idea of statistical physics to take into account the medium influence without immersion in the microscopic picture got specific development in the method. System evolution is described with some macroscopic parameters and we may vary their choice. In the case of boson variable elimination, the information about light generation becomes very restricted, and the scheme of reduced description allows saving field parameters in the consideration. At first the known results concerning the Dicke model were reproduced [14]. Since we were interested in emitter subsystem behavior, atomic operators of quasispin nature were used as parameters for reduced description. The most complete picture of statistical system evolution is provided by its statistical operator, so the above mentioned "Master equation" was constructed for the emitter subsystem. Then sets of equations for correlation functions of $r_i^{\pm,z}$ operators (i.e. collective operators of emitter subsystem) were obtained [15, 16] and we could speak about excitations in the quasispin complex (excitons). Their solutions remained unstudied in view of technical difficulties. The corresponding problem was considered in 1988 for the concentrated Dicke system – the set of six differential equations was solved numerically and it was regarded as success [17]. But the real interest was caused by systems with different spatial distribution of emitters. In 2008 the problem of field state in superradiant process was put forward [18]. It proved to be possible through introducing additional reduced description parameters concerning electromagnetic field. We put down the interaction Hamiltonian (4) in the form

$$H_{mf} = -\int d\mathrm{x} E_l^t(\mathrm{x}) P_l(\mathrm{x}) . \qquad (7)$$

where E_l^t is a macroscopic transversal field strength operator expressed via boson operators in a standard way for the spatial point with the radius-vector **x**, P_l is an operator of emitter electric polarization density taking into account the quasispin description of emitters $P_l(\mathrm{x}) = 2\sum_i d_{il}\, r_i^x \delta(\mathrm{x} - \mathrm{x}_i)$ where d_{il} stands for the matrix element of the working transition. Emitter subsystem state is described by the operator of power density $\varepsilon(\mathrm{x}) = \hbar\omega \sum_i r_i^z \delta(\mathrm{x} - \mathrm{x}_i)$. Notice that the operators of field strengths E_l^t, B_l, and their binary correlators as well as the operator $\varepsilon(\mathrm{x})$ satisfy the Peletminskii-Yatsenko conditions [12] and can form the basis for the reduced description of the process with their average values. In this case the statistical operator at great times turns to the quasiequilibrium form and provides the possibility of simple calculations of averages. Obviously, electromagnetic field as a statistical system can be approximately described by correlation functions of different orders at each time moment and the minimal order that is necessary for the statistical picture is the second one [19]. The correct approach to the field state description lies in using the simultaneous correlation functions built with various components of field vectors. Famous Glauber correlation functions measured in the coincidence scheme [20] should be considered as a tool for field state diagnostics. The connection between simultaneous and Glauber functions can be established due to the known time dependence of field modes.

We have built a set of evolution equations including those for binary correlation functions for the Dicke process for an arbitrary system of emitters (any shape, any distribution in space, any orientation; frequency spread is possible) [21]. For such a medium some ideas of electrodynamics of continuous media were applied. Normal waves have been investigated and the possibility of waves of correlations has been pointed out [22]. Now a new program of studying the Dicke model in the reduced description approach is considered. If we are interested in revealing the course of the process and the peculiarities of correlation arising, the only way to obtaining the information is the direct solving of the set of equations. Now we should pay our attention to the fact that we stay in the frame of the macroscopic approach and there is no necessity of simulating a separate emitter. Our equations operate with macroscopically averaged fields and correlations. If we know their values on a certain spatial lattice, the full picture of the process becomes clear. Of course, if the number of lattice sites makes one thousand, the number of correlation functions reaches several millions, but modern computer technology can meet the challenge. Experience of numerical simulations at our Theoretical Physics Department shows the feasibility of computer modeling of the Dicke process with field variables. At first stages we can restrict ourselves with less number of sites and then apply technological achievements. Such program has been presented this year at international conferences [23-25] and met positive attitude. There were presentations with similar ideas basing on the enormous technology capacity expansion [26]. For our proposal concerning Dicke model investigation, we see the possibility of obtaining the information about the way of correlation development and, in this connection, the analysis of the probable observation of non-classical field states in the system under consideration.

5. Conclusions

The Dicke model played an important role in studies of cooperative processes. Its history was full of unexpected turns and came to the present-day position of the reliable tool for analyzing the processes of self-organizing in nonequilibrium systems. Many theoretical methods were tested on the Dicke model. Interest to it is not exhausted: the Dicke system can be useful as an element of experimental complex for the implementation of new phenomena: Bose-Einstein condensate coupling with a cavity mode, ion trapping, quantum entanglement and so on (see [27] and references in it). Investigations of the Dicke model at Dnipropetrovsk National University on the basis of Bogolyubov reduced description method have brought a number of achievements. The possibility of studying the field evolution seems to be the most interesting among them. Previously the delay time of the superradiant pulse was fixed and controlled and now it becomes possible to obtain and analyze the whole picture of arising and development of correlations in the field. Incidentally the problem of generation field states with interesting correlation properties in the system of two-level emitters interacting via field should find its solution. Such program can be put into life on the basis of numerical simulation of the processes in the system with modern computer technology.

Acknowledgment

The author is grateful to Professor A. I. Sokolovsky for a fruitful cooperation in the Dicke model investigation and multiple discussions of the issues raised.

References

1. **Dicke, R. H.** Coherence in spontaneous radiation processes [Text] / R. H. Dicke //Phys. Rev. – 1954. – Vol. 93. – P. 99 – 110.

2. **Bogolyubov, N. N. (ml.).** Sverkhizlucheniye [Text] / N. N. Bogolyubov (ml.), A. S. Shumovsky. – Dubna: OIYaI, 1987. – 88 p.

3. **Andreyev, A. V.** Kooperativnyye yavleniya v optike [Text] / A. V. Andreyev, V. I. Yemelyanov, Yu. A. Ilinskiy. – Moscow: Nauka, 1988. – 288 p.

4. **Scully, M.** Quantum Optics [Text] / M. Scully, M. Zubairy. – Cambridge University Press, 1997. – 630 p.

5. **Lyagushyn, S. F.** To the problem of the Hamiltonian form for a system of two-level atoms interacting with electromagnetic field [Text] / S. F. Lyagushyn, A. I. Sokolovsky // Visnyk Dnipropetrovskoho Universytetu. Seriya Fizyka, Radioelektronika. – 2015. – Vol. 23, No. 1, Issue 22. – P. 54 – 59.

6. **Bogolyubov, N. N.** Kvazisredniye v zadachakh statisticheskoy mekhaniki. Preprint OIYaI R-1451 [Text] / N. N. Bogolyubov. – Dubna: JINR, 1963. – 123 p., also: // Izbrannyye trudy v 3 tomakh. Tom 3. – Kiev: Naukova dumka, 1971. – P. 174 – 243.

7. **Hepp K.** On the superradiant phase transition for molecules in a quantized radiation field: the Dicke Maser Model [Text] / K. Hepp, E. Lieb // Annals of Physics. – 1973. – Vol. 76, No. 2. – P. 360 – 404.

8. **Bogolyubov, N. N. (Jr.).** A class of exactly soluble many-body Hamiltonians with the interaction of substance and boson field [Text] / N. N. Bogolubov (Jr.), V. N. Plechko // Physica A. – 1975. – Vol. 82, No. 2. – P. 163 – 194.

9. **Skribanowitz, N.** Observation of Dicke Superradiance in Optically Pumped HF Gas [Text] / N. Skribanowitz, I. P. Herman, J. C. MacGillivray, M. S. Feld // Phys. Rev. Lett. – 1973. – Vol. 30, No. 8. – P. 309 – 312.

10. **Bogolyubov, N. N.** Kineticheskoye uravneniye dlya dinamicheskoy sistemy vzaimodeystvuyushchey s fotonnym polem [Text] / N. N. Bogolyubov, N. N. Bogolubov (ml.) // Fizika elementarnykh chastits i atomnogo yadra. – 1980. – Vol. 11, Issue 2. – P. 245 – 300.

11. **Bogolyubov, N. N. (Jr.).** Kinetic equation for a two-level system interacting with the electromagnetic field [Text] / N. N. Bogolyubov (Jr.), Fam Le Kien, A. S. Shumovsky // Theoret. and Math. Phys. – 1982. – Vol. 52, No. 3. – P. 893 – 898; also Preprint OIYaI P17-81-456. – Dubna, OIYaI, 1981.

12. **Akhiezer, A. I.** Metody statisticheskoy fiziki [Text] / A. I. Akhiezer, S. V. Peletminskiy. – M.: Nauka, 1977. – 367 p.

13. **Lyagushyn, S. F.** To kinetics of quantum systems in a medium / S. F. Lyagushyn, A. I. Sokolovsky, S. A. Sokolovsky, V. V. Yarlik // Visnyk Dnipropetrovskoho Universytetu. Seriya Fizyka, Radioelektronika. – 2016. – Vol. 24, No. 1, Issue 23. – P. 17 – 28.

14. **Lyagushyn, S. F.** Osnovne kinetychne rivnyannya dlya nadvyprominyuvalnoyi systemy u modeli Dicke [Text] / S. F. Lyagushyn, Yu. M. Salyuk, O.Y. Sokolovsky // Visnyk Dnipropetrovskoho Universytetu. Seriya Fizyka, Radioelektronika. – 2003. – Vol. 11, No. 1, Issue 10. – P. 44 – 49.

15. **Lyagushyn, S. F.** Efektyvnyy hamiltonian nadvyprominyuvalnoyi systemy [Text] / S. F. Lyagushyn, Yu. M. Salyuk, O.Y. Sokolovsky // Visnyk Dnipropetrovskoho Universytetu. Seriya Fizyka, Radioelektronika. – 2004. – Vol. 12, No. 1, Issue 11. – P. 75 – 83.

16. **Lyagushyn, S. F.** Kinetics of Emitters of Electromagnetic Field in the Bogolyubov Reduced Description Method [Text] / S. F. Lyagushyn, Yu. M. Salyuk, A. I. Sokolovsky // Physics of Particles and Nuclei. – 2005. – Vol. 36, Issue Suppl. 1. – P. S123 – S127.

17. **Bogolyubov, N. N. (ml.).** Primeneniye metoda isklyucheniya bozonnykh peremennykh k analizu evolutsii nekotorykh modelmykh sistem, vzaimodeystvuyushchikh s bozonnymi polyami. JINR Communication P17-88-4 [Text] / N. N. Bogolubov (ml.), S. F. Lyagushyn. – Dubna: OIYaI, 1988. – 12 p.

18. **Lyagushyn, S. F.** Correlation properties of electromagnetic field generated by emitters with random orientation [Text] / S. F. Lyagushyn, Yu. M. Salyuk, A. I. Sokolovsky // Proceedings of the 12th International Conference on Mathematical Methods in Electromagnetic Theory (Odesa, Ukraine, June 30 – July 2, 2008). – Odesa, 2008. – P.271 – 273.

19. **Lyagushyn, S.** Description of Field States with Correlation Functions and Measurements in Quantum Optics [Text] / S. Lyagushyn, A. Sokolovsky // Quantum Optics and Laser Experiments / Ed. S. Lyagushyn. – Croatia, Rijeka: InTech, 2012. – P. 3 – 24.

20. **Kilin, S. Ya.** Kvantovaya optika. Polya i ikh detektirovaniye [Text] / S. Ya. Kilin. – Russia, Moscow: Editorial URSS, 2003. – 176 p. – ISBN 5-354-00442-X.

21. **Lyagushyn, S. F.** Kinetics of system of emitters and nonequilibrium electromagnetic field [Text] / S.F. Lyagushyn, A.I. Sokolovsky // Physics of Elementary Particles and Nuclei. – 2010. – Vol. 41, No. 7. – P. 1035 – 1038.

22. **Lyagushyn, S.** Electromagnetic waves in medium consisting of two-level emitters [Text] / S. Lyagushyn, Yu. Salyuk, A. Sokolovsky // Proceedings of 2012 International Conference on Mathematical Methods in Electromagnetic Theory (Kharkiv, Ukraine, August 28-30, 2012), Proceedings CD-ROM. – P. 205 – 208. – ISBN 978-1-4673-4479-1.

23. **Lyagushyn, S. F.** To the problem of correlation properties of light generated in superradiance processes [Electronic resource] / S. F. Lyagushyn, A. I. Sokolovsky // Bogolyubov Conference "Problems of Theoretical Physics" dedicated to the 50th anniversary of the Bogolyubov Institute for Theoretical Physics of the NAS of Ukraine (Kyiv, Ukraine, May, 24 – 26, 2016), Program & Abstracts. – P. 22. – http://50anniversary.bitp.kiev.ua /static/abstracts.pdf (Abstract O11).

24. **Lyagushyn, S. F.** Superradiance in Dicke systems: a picture with field correlation functions [Electronic resource] / S. F. Lyagushyn, A. I. Sokolovsky // 16th International Conference on Mathematical Methods in Electromagnetic Theory (Lviv, Ukraine, July, 5 – 7, 2016), DVD Conference Proceedings. – P. 369 – 372. – ISBN 978-1-5090-1956-4.

25. **Lyagushyn, S. F.** To the problem of field macrostate description and investigation / S. F. Lyagushyn, A. I. Sokolovsky [Electronic resource] // 2016 IEEE 13th International Conference on Laser & Fiber-Optical Networks Modeling (LFNM), CD Conference Proceedings. – Odessa, Ukraine, 13-15 September 2016. – P. 74–76. – ISBN 978-1-5090-1009-7.

26. **Nosich, A. I.** Essentials and Merits of the Method of Analytical Regularization in Computational Optics and Photonics [Electronic resource] / A. I. Nosich, M. V. Balaban, V. O. Byelobrov, D. M. Natarov, O. V. Shapoval // 2016 IEEE 13th International Conference on Laser & Fiber-Optical Networks Modeling (LFNM), CD Conference Proceedings. – Odessa, Ukraine, 13-15 September 2016. – P. 77 – 81. – ISBN 978-1-5090-1009-7.

27. **Garraway, B. M.** The Dicke model in quantum optics: Dicke model revisited [Text] // B. M. Garraway // Phil. Trans. R. Soc. A – 2011. – Vol. 369. – P. 1137 – 1155.

Received 01.12.2016