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T-REGIONS IN REISSNER-NORDSTRÖM SPACETIMES WITH VACUUM ENERGY

T-regions in spherically symmetric spacetime around an electrically charged distribution of matter in presence of vacuum energy are investigated. The solution of Einstein field equations is derived and examined for a constant vacuum energy density. This solution is asymptotically de Sitter generalization of Reissner-Nordström solution. The conditions for T-regions defined by mass function are studied. It is shown that in dependence on ratio of the vacuum energy density and the total mass of source the different types of spacetime are possible. When the mass of the source exceeds the upper limit $M_{\max}$ the scheme for T-regions and R-regions is R-T. There is no black hole-like spacetimes possible in this case. When the mass of the source is less then the $M_{\max}$ the scheme for T-regions and R-regions is defined by the electric charge of the source. It may be R-T (no black hole), R-T-R-T (charged black hole), R-R-T and R-T-T (extreme cases with horizons). Charged black holes in presence of vacuum energy have the upper limit on charge that is slightly bigger then one for Reissner-Nordström solution. In case when the mass of the source is in range 0.5 $M_{\max} < M < M_{\max}$ there is the low limit on electric charge of black hole.

Keywords: Einstein equations, T-regions, vacuum energy density, Reissner-Nordström-de Sitter solution, charged black hole.

1. Introduction

We consider a spherically symmetric spacetime around an electrically charged distribution of matter. The gravitational field of such sources in empty space is described by well-known Reissner-Nordström solution: [1]

$$ds^2 = \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)dt^2 - \left(1 - \frac{2M}{r} + \frac{q^2}{r^2}\right)^{-1} dr^2 - r^2\left(d\theta^2 + \sin^2\theta \cdot d\varphi^2\right) \qquad (1)$$

where M is the total mass of the source, q is the electric charge, and r is the distance to central source.

The metric coefficient $g_{\theta\theta}$ is in general a function of both time t and radial R coordinates. But there may be so called T-regions in spacetime for which $g_{\theta\theta}$ can be written as a function of time coordinate only. Inside T-region the distance r to the center can not be constant for any test particle. So the inner region of black hole is the T-region separated from outer R-region by event horizon.

For Reissner-Nordström solution the T-region exists only if $|q|<M$. Under this condition the central source is surrounded by inner R-region, then there is the T-region embedded by the outer R-region (scheme R-T-R). The horizons are located at [2]

$$r_{1,2} = M \pm \sqrt{M^2 - q^2}\ . \qquad (2)$$

For sources with $q \to M$ the inner R-region and the outer one are closed up, so there is no T-region at all (extreme case). Therefore charged black holes have the restriction on their electric charge:

$$q < M \; . \tag{3}$$

Recent cosmological observations indicate that the empty space has a nonzero energy (so called dark energy). The vacuum energy density ε_v is very small but nonzero. The latest observation data put restriction on the dark energy equation of state as [3] $p_v = w\varepsilon_v$, $w = -1.019^{+0.075}_{-0.080}$ where p_v is vacuum "pressure" and ε_v is vacuum energy density. The simplest interpretation of vacuum energy is a positive cosmological constant: $\varepsilon_v = \mathrm{Const}$, $p_v = -\varepsilon_v$.

For a non-charged spherically symmetric source in presence of vacuum energy spacetime always contains the outer T-region (solution is asymptotically de Sitter). So for charged black holes with vacuum energy the restrictions on their mass and electric charge are different from ones without vacuum energy. Some properties of the so called Reissner-Nordström-de Sitter black holes were discussed in works [4-7].

The aim of the present paper is to analyze scheme for T-regions and R-regions and to investigate how the presence of vacuum energy influences on the characteristics of charged black holes.

2. Field equations and solution

In case of spherically symmetric empty spacetime the line element can be written as

$$ds^2 = e^{\nu} dt^2 - e^{\lambda} dR^2 - r^2 \left(d\theta^2 + \sin^2 \theta \cdot d\varphi^2 \right) \tag{4}$$

where ν and λ are functions of both radial R and time t coordinates, θ and φ are spherical angles. We use geometrized units $c = G = 1$.

For T-region the metric coefficient $g_{\theta\theta} = r^2$ is assumed to be the function of time coordinate t only, so we can put

$$r^2 = t^2 . \tag{5}$$

The energy-momentum tensor T^{μ}_{ν} for spacetime under consideration consists of two parts: for a vacuum and for an static electric field. In chosen coordinates T^{μ}_{ν} is diagonal with

$$T^0_0 = \varepsilon_v + \frac{E^2}{8\pi} , \; T^1_1 = \varepsilon_v + \frac{E^2}{8\pi} , \; T^2_2 = T^3_3 = \varepsilon_v - \frac{E^2}{8\pi} \tag{6}$$

where $\varepsilon_v = \mathrm{Const}$ is vacuum energy density, E is electric field intensity.

The Einstein field equations for metric (4), (5) take the form

$$8\pi T^0_0 = \left(e^{-\nu} - 1\right)\frac{1}{t^2} + e^{-\nu}\frac{\dot{\lambda}}{t} , \tag{7}$$

$$8\pi T^1_1 = \left(e^{-\nu} - 1\right)\frac{1}{t^2} - e^{-\nu}\frac{\dot{\nu}}{t} , \tag{8}$$

$$8\pi T^2_2 = \frac{e^{-\nu}}{4}\left(2\ddot{\lambda} + \dot{\lambda}^2 + \frac{2}{t}\left(\dot{\lambda} - \dot{\nu}\right) - \dot{\lambda}\dot{\nu} \right) , \tag{9}$$

$$\frac{\partial \nu}{\partial R} = 0 \tag{10}$$

where the dot denotes differentiation with respect to t.

For the spacetime under consideration $T_1^1 = T_0^0$, so from (7), (8) we have $e^{\lambda} = e^{-\nu(t)} f_1(R)$. We may put the arbitrary function $f_1(R)$ to 1, so

$$e^{\lambda(t)} = e^{-\nu(t)}. \tag{11}$$

The mass function for the line element (4), (5) takes the form [8]:

$$m(t) = t\left(1 + e^{-\nu}\right). \tag{12}$$

Equations (9) – (12) may be written as

$$e^{\lambda} = e^{-\nu} = \frac{m}{t} - 1, \tag{13}$$

$$\dot{m} = \left(8\pi\varepsilon_{\text{v}} + E^2\right)t^2, \tag{14}$$

$$\frac{d\left(8\pi\varepsilon_{\text{v}} + E^2\right)}{E^2} = -\frac{4}{t}dt. \tag{15}$$

The solution for T-regions in spacetime under consideration is

$$ds^2 = \left(\frac{m}{t} - 1\right)^{-1} dt^2 - \left(\frac{m}{t} - 1\right)dR^2 - t^2\left(d\theta^2 + \sin^2\theta \cdot d\varphi^2\right), \tag{16}$$

$$E = \frac{q}{t^2}, \quad m(t) = \frac{1}{3}\cdot 8\pi\varepsilon_{\text{v}} t^3 - \frac{q^2}{t} + 2M \tag{17}$$

where M is the total mass of the source, q is the source charge.

The solution (16), (17) describes the T-region of Schwarzschild spacetime if $q \to 0$, $\varepsilon_{\text{v}} \to 0$. If $\varepsilon_{\text{v}} \to 0$, $q \neq 0$ then the solution (16), (17) reduces to the one for T-region of Reissner-Nordström solution. The solution (16), (17) is asymptotically de Sitter (for $t \to \infty$) so it is often called Reissner-Nordström-de Sitter solution.

3. Conditions for T-regions in Reissner-Nordström spacetimes with vacuum energy

For T-region in spacetime described by the line element (4) metric coefficients e^{ν}, e^{λ} are positive. Thus for T-regions the mass function must satisfy the condition

$$\frac{m}{t} - 1 > 0. \tag{18}$$

For mass function (17) this lead to condition

$$\frac{1}{3}\cdot 8\pi\varepsilon_{\text{v}} t^2 - \frac{q^2}{t^2} + \frac{2M}{t} - 1 > 0. \tag{19}$$

To investigate the condition (19) we rewrite it in following form:

$$\frac{1}{t^2 r_c^2}\left(f(t)-q^2 r_c^2\right)>0 \tag{20}$$

where

$$f(t)=t^4-r_c^2 t^2+2Mr_c^2 t\ ,\ r_c=\left(\frac{1}{3}\cdot 8\pi\varepsilon_v\right)^{-1/2}. \tag{21}$$

T-regions correspond to the coordinates t for which

$$f(t)>q^2 r_c^2 . \tag{22}$$

Condition $f(t_h)=q^2 r_c^2$ defines the location t_h of horizons that separate regions.

Behavior of function $f(t)$ for positive t depends on ratio of the vacuum energy density ε_v and the total mass of the source M.

In the case (a) when

$$\varepsilon_v>2\left(8\pi\cdot 9M^2\right)^{-1},\ r_c<3\sqrt{3/2}M\ , \tag{23}$$

function $f(t)$ is monotone increasing from $f(0)=0$.

In the case (b) when

$$\varepsilon_v=2\left(8\pi\cdot 9M^2\right)^{-1},\ r_c=3\sqrt{3/2}M\ , \tag{24}$$

function $f(t)$ has an inflection point at $t_0=3M/2$, and $f(t_0)=3(3M/2)^4>0$.

In the case when

$$\varepsilon_v<2\left(8\pi\cdot 9M^2\right)^{-1},\ r_c>3\sqrt{3/2}M\ , \tag{25}$$

function $f(t)$ has two extremum points at $t_{(+)}$ and $t_{(-)}$ where

$$t_{(\pm)}=\sqrt{\frac{2}{3}}r_c\cos\left(\frac{\pi}{3}\pm\frac{1}{3}\arccos\left(\frac{3M}{r_c}\sqrt{\frac{3}{2}}\right)\right). \tag{26}$$

Note that $t_{(+)}\le t_{(-)}$. At first extremum point $f(t_{(+)})>0$ for any $\varepsilon_v>0$. At second extremum point value of function is $f(t_{(-)})=r_c^2 t_{(-)}(3M-t_{(-)})/2$.

The value $f(t_{(-)})$ is: $f>0$ for $\left(8\pi\cdot 9M^2\right)^{-1}<\varepsilon_v<2\left(8\pi\cdot 9M^2\right)^{-1}$ (case (c)), $f=0$ for $\varepsilon_v=\left(8\pi\cdot 9M^2\right)^{-1}$ (case (d)) and $f<0$ for $\varepsilon_v<\left(8\pi\cdot 9M^2\right)^{-1}$ (case (e)).

The plot of the function $(r_c)^4 f(t)$ for different cases is presented at Fig. 1. For the cases (a) – (e) there are different schemes for T-regions and R-regions in spacetime. The condition (22) defines T-regions. It can be shown on the plot of $f(t)$ that the ranges of coordinate t where curve $r_c^{-4} f(t)$ goes over value $q^2 r_c^{-2}$ are T-regions. In the cases (a) and (b) there is the only possible scheme: R-T. The central R-region is surrounded by outer T-region for any value of charge q.

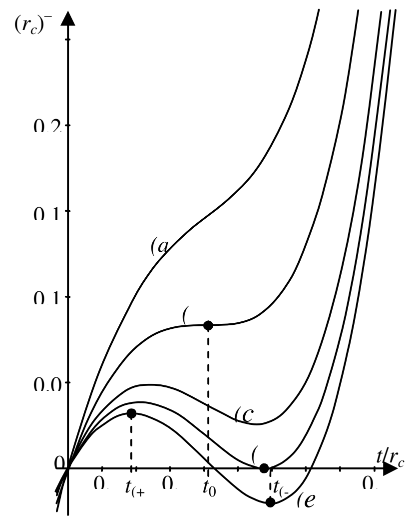

Fig. 1. Behavior of function *f(t)* for positive *t*. Curves (*a*) – (*e*) corresponds for cases with different values of parameter $2M/r_c$. Case (*a*): $2M/r_c = 0.7$. Case (*b*): $2M/r_c = (2/3)^{3/2} \approx 0.544331$. Case (*c*): $2M/r_c = 0.43$. Case (*d*): $2M/r_c = 2/(3^{3/2}) \approx 0.3849$. Case (*e*): $2M/r_c = 0.35$.

In the case (*c*) the scheme for T-regions and R-regions depends on the ratio of values $f\left(t_{(+)}\right)$, $f\left(t_{(-)}\right)$ and $q^2 r_c^2$.

If (i) $q^2 r_c^2 > f\left(t_{(+)}\right) = r_c^2 t_{(+)} \left(3M - t_{(+)}\right)/2$ then the scheme is R-T (Fig. 2).

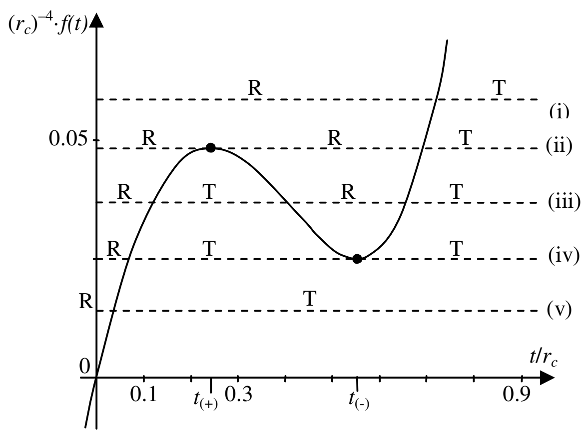

Fig. 2. The scheme for T-regions and R-regions in case (*c*): $2M/r_c = 0.43$. Dashed lines (i) – (v) correspond to values of $q^2(r_c)^{-2}$.

If (ii) $q^2 r_c^2 = f\left(t_{(+)}\right) = r_c^2 t_{(+)} \left(3M - t_{(+)}\right)/2$ then the scheme is R-R-T. The central R-region is surrounded by another R-region. This R-regions are separated by horizon at $t = t_{(+)}$. In the case (iii) when

$$f\left(t_{(-)}\right) < q^2 r_c^2 < f\left(t_{(+)}\right) \tag{27}$$

there exist T-region between two R-regions (scheme is R-T-R-T). The event horizon occurs at $t_{(+)} < t < t_{(-)}$ and the solution (16), (17) describes the charged black hole.

In the case (iv) when $q^2 r_c^2 = f\left(t_{(-)}\right)$ the scheme is R-T-T: inner and outer T-regions are joined. If (v) $q^2 r_c^2 < f\left(t_{(-)}\right)$ then the scheme is R-T.

In the case (*d*) the similar analysis can be conducted. Note that $f\left(t_{(-)}\right) = 0$ in this case. So if $q^2 r_c^2 > f\left(t_{(+)}\right)$ the scheme is R-T. For $q^2 r_c^2 = f\left(t_{(+)}\right)$ the scheme is R-R-T. And if $q^2 r_c^2 < f\left(t_{(+)}\right)$ then the scheme is R-T-R-T (charged black hole).

In case (*e*) there are three possible schemes: R-T if $q^2 r_c^2 > f\left(t_{(+)}\right)$, R-R-T if $q^2 r_c^2 = f\left(t_{(+)}\right)$ and R-T-R-T if $q^2 r_c^2 < f\left(t_{(+)}\right)$.

In the case of non charged source the condition of T-region (22) reduces to $f(t) > 0$. It can be shown from Fig. 1 that there is the central T-region for all values of ε_v.The R-region exists only if $\varepsilon_v < \left(8\pi \cdot 9M^2\right)^{-1}$. The schemes are: T (only T-region) for $\varepsilon_v > \left(8\pi \cdot 9M^2\right)^{-1}$; T-T for $\varepsilon_v = \left(8\pi \cdot 9M^2\right)^{-1}$; and T-R-T for $\varepsilon_v < \left(8\pi \cdot 9M^2\right)^{-1}$.

In the case of charged source with zero vacuum energy (Reissner-Nordström solution) we have $r_c \to \infty$. Rewriting condition (19) as $2Mt - t^2 > q^2$ we find that the schemes are: R (only R-region) for $q^2 > M^2$; R-R for $q^2 = M^2$; and R-T-R for $q^2 < M^2$ (charged black hole).

4. Conclusions

In case of charged spherically symmetric sources in presence of vacuum energy different types of spacetime are realized depending on the ratio of values ε_v, *M* and *q*. For all cases there are the central R-region around the source and the outer T-region. The outer T-region is separated from inner regions by the cosmological horizon as in de Sitter solution.

For a black hole-like spacetime there must be the inner T-region and the outer R-region separated by event horizon (scheme R-T-R-T). For a fixed value of vacuum energy density ε_v the spacetimes of such type have restriction on the mass *M* and electric charge *q* of source.

The mass of the charged black hole must not exceed the limit

$$M < M_{\max} = \frac{\sqrt{2}}{3\sqrt{3}} r_c, \quad r_c = \left(\frac{1}{3} \cdot 8\pi\varepsilon_v\right)^{-1/2}. \tag{28}$$

For extreme case $M \to M_{\max}$ (Fig. 1, curve (*b*)) the scheme is R-T.

The electric charge *q* of the black hole must satisfy conditions

$$\begin{aligned} &q_1 < q < q_2 \quad \text{if} \quad \frac{1}{\sqrt{2}} M_{\max} < M < M_{\max}, \\ &q < q_2 \quad \text{if} \quad M \le \frac{1}{\sqrt{2}} M_{\max} \end{aligned} \tag{29}$$

where the limits q_1 and q_2 are

$$(q_1)^2 = \frac{1}{2} t_{(-)} \left(3M - t_{(-)}\right), \ (q_2)^2 = \frac{1}{2} t_{(+)} \left(3M - t_{(+)}\right) \tag{30}$$

and $t_{(-)}$, $t_{(+)}$ are defined by (26). Note that the upper limit on charge $M < q_2 < 1.125M$ differ from the one for Reissner-Nordström solution (3). For extreme case $q \to q_2$ (Fig. 2, line (ii)) the scheme is R-R-T, for extreme case $q \to q_1$ (Fig. 2, line (iv)) the scheme is R-T-T. If conditions (28), (29) are not satisfied then the scheme is R-T.

So for charged black holes with vacuum energy the main differences from the case $\varepsilon_v = 0$ are:

1) the upper limitation on mass of black hole (28);

2) the upper limit on electric charge (29) is bigger than one for Reissner-Nordström solution;

3) for black holes of mass $1/\sqrt{2}\, M_{\max} < M < M_{\max}$ there is the low limit on electric charge $q_1 < q < q_2$.

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