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THREE-PHOTON VERTEX IN DENSE FERMIONIC MEDIUM WITH STRONG FIELD

Investigation of the properties of the tensor three-photon vertex in the presence of strong fields and dense medium is accomplished in this article. Calculations of the tensor function in a general form and its exact value for the case of strong-field approximation and a small effective fermion mass are performed. Also, an exact calculation is made for the spectrum of photon scattering into longitudinal and transverse modes in the case of orthogonal momentum of the initial photon and field lines of the external field. The calculations are carried out with using the perturbation theory approach ‒ the tensor function is represented as a sum of Feynman diagrams in the first order of perturbation theory. The processes of photon scattering in external fields corresponding to certain elements of the vertex function are considered. Both elastic photon scattering in magnetic field caused by the presence of a chemical potential of the medium and photon splitting into longitudinal and transverse components caused by the joint effect of the presence of the chemical potential and a strong magnetic field are studied in detail.

Keywords: three-photon vertex, vertex function, dense medium, photon in medium, c-parity violation.

Проводиться дослідження властивостей тензора трьохфотонної вершини в присутності сильних полів і густого середовища. Виконується обчислення тензорної функції в загальному вигляді та розрахунок точних її значень для випадку наближення сильного поля та малої ефективної маси ферміона. Також проводиться точний розрахунок спектра розсіювання фотонів на поздовжню та поперечну моди для випадку ортогональності імпульсу початкового фотона й силових ліній зовнішнього поля. Обчислення проводяться з використанням підходу теорії збурень ‒ тензорна функція представлена як сума фейнманівських діаграм першого порядку теорії збурень. Розглянуто ті процеси розсіювання фотонів у зовнішніх полях, які відповідають певним елементам вершинної функції. Детально досліджені як пружне розсіяння в магнітному полі, викликане наявністю хімічного потенціалу середовища, так і розщеплення на поздовжню і поперечну компоненти, викликане ефектом спільної присутності хімічного потенціалу та сильного магнітного поля.

Ключові слова: трьохфотонна вершина, вершинна функція, густе середовище, фотони в середовищі, порушення с-парності.

Проводится исследование свойств тензора трёхфотонной вершины в присутствии сильных полей и плотной среды. Выполняется вычисление тензорной функции в общем виде и расчёт точных её значений для случая приближения сильного поля и малой эффективной массы фермиона. Также производится расчёт спектра рассеяния фотонов на продольную и поперечную моды для случая ортогональности импульса начального фотона и силовых линий внешнего поля. Вычисления проводятся с использованием подхода теории возмущений – тензорная функция представлена как сумма фейнмановских диаграмм первого порядка теории возмущений. Рассмотрены те процессы рассеяния фотонов во внешних полях, которые соответствуют определённым элементам вершинной функции. Детально исследованы как упругое рассеяние в магнитном поле, вызванное наличием химического потенциала среды, так и расщепление на продольную и поперечную компоненты, вызванное эффектом совместного присутствия химического потенциала и сильного магнитного поля.

Ключевые слова: трёхфотонная вершина, вершинная функция, плотная среда, фотоны в среде, нарушение с-чётности.

1. Introduction

The problem of describing the behavior of free photons and electromagnetic fields in dense fermionic medium is still relevant today, especially when it comes to processes in strong fields. The presence of a medium causes a violation of c-parity, which in turn leads to the possible existence of multiphoton vertices with an odd number of external photon lines.

Investigation of real nonlinear processes in dense medium frequently faces the problem of medium "purity" since there are powerful magnetic fields – up to 10^{15} T – and spontaneous creation of chromomagnetic field can occur in the medium during its creation, in addition to chemical potential.

To consider such cases, we must calculate the vertex function generated by the presence of strong fields in a dense medium. However, namely the set of factors operating in the medium can give the most clear "signals" of medium creation and allow investigating its properties.

In this paper a three-photon vertex in a dense medium and in the presence of strong fields is studied. With this aim in view, the tensor corresponding to the three-photon vertex is considered with using the perturbation theory, i.e. as a sum of three-photon vertex in a medium and three four-photon vertexes with added external field lines. For special cases, the exact expressions of the corresponding nonzero tensor components are given.

2. Tensor of three-photon vertex in dense medium

The goal of this section is getting the nonzero tensor components of three-photon vertex in a medium that exists for the case of one-loop approximation at rest for zero temperature. Under these conditions the tensor is defined by the following integral

$$\Pi_{\mu\nu\gamma}(k,k',k'')=\delta(k+k'+k'')\frac{e^3}{(2\pi)^3}\int dp^4(\gamma_\mu G(p+k)\gamma_\nu G(p-k')\gamma_\gamma G(p)+ \\ +\gamma_\mu G(p)\gamma_\nu G(p+k')\gamma_\gamma G(p-k')) \tag{1}$$

μ, ν, and γ are indices that run values from one to four, and $G(p)$ is an electron Green's function

$$G(p)=\frac{-ip+m}{p^2+m^2} \tag{2}$$

recorded in the Euclidean metric, i.e.

$$p=\begin{cases} p_\rho, \rho=1,2,3 \\ p_4+i\mu, \rho=4 \end{cases} \tag{3}$$

μ – chemical potential of medium γ_μ – Dirac matrices.

Exact expressions for static fields [3] are obtained in calculations of the integral (1) when $k_4=0$. The form of vertex functions is greatly simplified in this case and the only nonzero tensor components are Π_{444} and Π_{ij4}, which, in the case of small momenta $\frac{k^{(1)}}{a} << 1, \frac{k^{(2)}}{a} << 1$ where $a = \sqrt{\mu^2 - m^2}$, in the symmetric case $k^{(1)}/k^{(2)} \to 1$ have the following asymptotic behavior:

$$\Pi_{444} = \frac{e^3}{\pi^3}\theta(\mu^2 - m^2)\sqrt{\mu^2 - m^2}(6 + \left(\sum_{n>l}^{3} \sum_{l=1}^{2} \frac{\beta_{in} - \pi q_{in}}{\sin\beta_{in}} \begin{pmatrix} (2(1+\cos\beta_{12})^{\frac{2-\beta}{2}} + \\ +(3-n)(1+\cos\beta_{12}) - \cos\beta_{in}) \end{pmatrix} \right) + O(\frac{k^{(1)}}{a}) \tag{4}$$

$$\Pi_{4ij} = \frac{e^3}{\pi^3}\theta(\mu^2 - m^2)\sqrt{\mu^2 - m^2}(\delta_{ij} + \cos\alpha_{1i}\cos\alpha_{3j}\cos\beta_{13} - -\cos\alpha_{1i}\cos\alpha_{1j} - \cos\alpha_{3i}\cos\alpha_{3j}) + O(\frac{k^{(1)}}{a})$$

where β_{in} is an angle between the vectors k^l and k^n, $\cos(\alpha_{1i})$ – direction cosines of the vector k^1, q_{in} – random integers that satisfy the equality $q_{12} + q_{13} + q_{23} = 2$, $\theta(\mu^2 - m^2)$ – Heaviside function.

In the case of large momenta $\frac{k^{(1)}}{a}, \frac{k^{(2)}}{a} >> 1$ in the symmetric limit, the tensor components are as follow:

$$\Pi_{444} = O(\frac{k^{(1)}}{a}) \ , \ \Pi_{4ij} = O(\frac{k^{(1)}}{a}) \ . \tag{5}$$

This result shows that approximation of large momenta, and, as a result, short distances or strong fields, the medium becomes asymptotically transparent, and its effect on the properties of photons with a great momentum is small. This is because the value of the vertex function is linearly proportional to the chemical potential, which in the case of large momentum is a small parameter.

3. Three-photon tensor in long-wave approximation

For low frequency photons, and therefore small values of $\mathbf{k}_4$, while using the linear approximation, the tensor splits into three group of summands, whereby the coefficient multipliers at them have the values H_1, H_2, H_3 [5], presented in Appendix.

Using these approximate values of the factors, we find the expressions for the elements of the tensor F_{444} that is associated with Π_{444} by the equation $\Pi_{444} = 2(e/2\pi)^3 \int d^3 p F_{444}$

Having the exact expression for static fields, we can consider its contribution, and then we can calculate those terms containing $\mathbf{k}_4$ in the first degree

$$F_{444} \approx F_{444}^{\;stat.} + \mathrm{Re}\left\{ \begin{array}{l} H_1\left[\mathbf{pk}^{(2)}k_4^{(1)} + \left(\mathbf{pk}^{(1)} - \varepsilon_0^2\right)k_4^{(2)}\right] + \\ H_2\left[\left(\varepsilon_0^2 - \mathbf{pk}^{(1)} + 2\varepsilon_1^2\right)k_4^{(1)} + \left(2\varepsilon_0^2 - \mathbf{pk}^{(2)} - \mathbf{k}^{(1)}\mathbf{k}^{(2)} + \varepsilon_1^2\right)k_4^{(2)}\right] + \\ H_3\left[\left(\varepsilon_0^2 - \mathbf{pk}^{(1)} + \varepsilon_2^2\right)k_4^{(1)} + \left(2\varepsilon_0^2 - \mathbf{pk}^{(2)} - \mathbf{k}^{(1)}\mathbf{k}^{(2)} + 2\varepsilon_2^2\right)k_4^{(2)}\right] \end{array} \right\} \quad (6)$$

In the case of F_{ij4} we take into account that $i \neq 4, j \neq 4$, i.e. $\delta_{i4} = \delta_{j4} = 0$, and the used approach is defined as $k_4^{(a)}k_4^{(b)} = O(k_4), a, b \in 1, 2$, so the tensor element is simplified significantly.

Similarly, we perform the conversion to other elements of the tensor, taking into account that in the static case $F_{44i}^{\;stat} = F_{ijl}^{\;stat} = 0$ and the expression for F_{ijl} F_{ij4}, which are present in an explicit form in Appendix.

The found value of tensor components indicates that the process of creation of real photons occurs only in varying fields, as appropriate tensor components has only the dynamic part. At the same time, the assumption of long-wave photons allows only magnetic field fission into real photons.

In the case of a real photon scattering in a magnetic field in the considered approximation, we can calculate explicitly the final expression for the effective cross-section

$$k_i^{(1)}k_j^{(2)}F_{ij\mu}\left(k_\mu^{(1)} + k_\mu^{(2)}\right) \approx -\mathrm{Im}\, k_4^{(1)}k^2 [\, 2(1+\cos\gamma)(\cos\alpha + \cos\beta) \times$$

$$\times \frac{n_e(\varepsilon_0) - n_p(\varepsilon_0)}{2i[2p\cos\alpha + k][-2p\cos\beta + k]}\, p + (k(1+\cos\gamma) + p(\cos\alpha + \cos\beta) \times$$
$$\times \left(\frac{n_e(\varepsilon_1) - n_p(\varepsilon_1)}{2i[2p\cos\alpha - k][-2p(\cos\beta + \cos\alpha)]} + \frac{n_e(\varepsilon_2) - n_p(\varepsilon_2)}{2i[2p\cos\beta - k][-2p(\cos\beta + \cos\alpha)]}\right)] \quad (7)$$

where cosγ is the cosine of the angle between the vectors and cosα, cosβ – of the angles between the vector p and k_1 and k_2 , correspondingly. With the general form of the expression, it implies that photon scattering cross section in a magnetic field is proportional to the photon wave vector in the second degree.

4. Three-photon vertex in a dense medium and strong fields

We can consider influence of the presence of a strong field in a medium, using the perturbation theory in one-loop approximation. As the addition to the "naked" three-photon diagram, we take three four-photon vertexes, each of which, in addition to three external photon lines, has a line of field interaction with fermions in the loops.

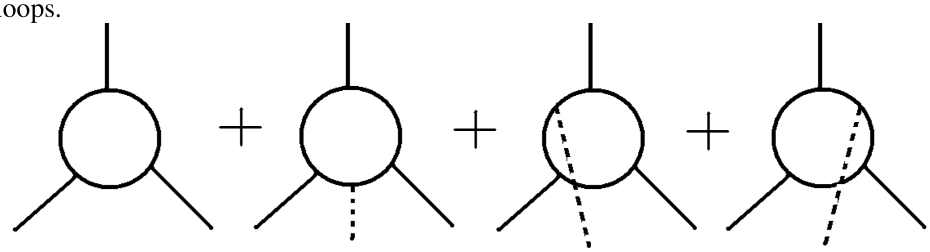

Fig. 1. The sum of Feynman diagrams of the first order of perturbation theory

Solid straight lines is the external photons, solid circle is the loop of fermions in the medium with the chemical potential, the dashed lines is the line of the external field interaction with fermions in the loop

General expression for the tensor $\Pi_{\mu\nu\gamma}\left(k,k',k''\right)$ three-photon vertex for this case is presented in Appendix.

Investigating this expression, we can calculate the cross section of photon decay as an exact solution using the approach of a strong field and a small effective mass:

$$\left|\mu^2-m^2\right| << \left(k_4\sin\theta\right)^2 << eB \tag{8}$$

where Θ is an angle between the direction of the initial photon momentum and the magnetic field lines.

For the used approach, cross section of photon splitting into transverse and longitudinal mod, will have the form of this precise equation:

$$W_{\Downarrow\to\Downarrow\perp}=\frac{\alpha^3 k_4\sin^2\theta}{16}\left(1-\frac{2\sqrt{\mu^2-m^2}}{k_4\sin\theta}\right)\left[1+2\left(1+\frac{2\sqrt{\mu^2-m^2}}{k_4\sin\theta}\right)^2\ln\left(1+\frac{2\sqrt{\mu^2-m^2}}{k_4\sin\theta}\right)-\right.$$
$$\left.-\frac{8\dfrac{\mu^2-m^2}{\left(k_4\sin\theta\right)^2}}{\left(1-\dfrac{2\sqrt{\mu^2-m^2}}{k_4\sin\theta}\right)}\left(\frac{2-4\dfrac{\mu^2-m^2}{\left(k_4\sin\theta\right)^2}}{\left(1-\dfrac{2\sqrt{\mu^2-m^2}}{k_4\sin\theta}\right)}\ln\left(\frac{k_4\sin\theta}{2\sqrt{\mu^2-m^2}}\right)-1\right)\right] \tag{9}$$

In the same approximation, the spectrum of the scattered photons can be calculated for the case when the initial photon momentum is orthogonal to the field

$$\frac{dW_{\Downarrow\to\Downarrow\perp}}{d\omega'}=\frac{\alpha^3\sqrt{\left(\omega-\omega'\right)^2-4\left(\mu^2-m^2\right)}}{2\left(\omega'+\sqrt{\left(\omega-\omega'\right)^2-4\left(\mu^2-m^2\right)}\right)} \tag{10}$$

where ω and ω' are energy of photons – initial and scattered, longitudinal mode, accordingly.

The presence of the chemical potential allows using the approach mentioned above for the case of low-energy photons and, therefore, the moderate values of the field in the medium.

5. Discussions

At the real dense medium creation, the strong fields will appear inside it; the presence of them can significantly distort the effects arising from the availability of the chemical potential in a "pure" medium, so we need to consider impact of additional factors.

At the same time, the consideration of effects, which are generated by a combination of strong fields and chemical potential, will help to describe some new processes that can occur in real-dense medium.

One of these processes is the process of splitting photons into the transverse and longitudinal mode in an interaction with magnetized medium. Thus, the presence of significant chemical potential in the medium allows to observe noticeable splitting even for low photon energies and not over-critical fields.

APPENDIX

Here we list expressions for the multipliers to three groups of summands in the vertex tensor

$$H_1 \approx \frac{n_e(\varepsilon_0) - n_p(\varepsilon_0)}{2i\varepsilon_0\left[2\mathbf{pk}^{(1)} + \left(\mathbf{k}^{(1)}\right)^2\right]\left[-2\mathbf{pk}^{(2)} + \left(\mathbf{k}^{(2)}\right)^2\right]} \quad ,$$

$$H_2 \approx \frac{n_e(\varepsilon_1) - n_p(\varepsilon_1)}{2i\varepsilon_1\left[2\mathbf{pk}^{(1)} - \left(\mathbf{k}^{(1)}\right)^2\right]\left[-2\mathbf{p}\left(\mathbf{k}^{(1)} + \mathbf{k}^{(2)}\right) + \left(\mathbf{k}^{(2)}\right)^2 - \left(\mathbf{k}^{(1)}\right)^2\right]},$$

$$H_3 \approx \frac{n_e(\varepsilon_2) - n_p(\varepsilon_2)}{2i\varepsilon_2\left[2\mathbf{pk}^{(1)} - \left(\mathbf{k}^{(2)}\right)^2\right]\left[-2\mathbf{p}\left(\mathbf{k}^{(1)} + \mathbf{k}^{(2)}\right) + \left(\mathbf{k}^{(1)}\right)^2 - \left(\mathbf{k}^{(2)}\right)^2\right]}$$

Below we give expressions for the elements of the tensor function

$$F_{i44} \approx \mathrm{Im}\left\{\begin{array}{l} H_2\varepsilon_1\left[\left(\left(k_i^{(1)} + 2p_i\right) - k_i^{(2)}\right)k_4^{(1)} + \left(k_i^{(1)} + 2p_i\right)k_4^{(2)}\right] + \\ H_3\varepsilon_2\left[\left(\left(k_i^{(2)} + 2p_i\right) - k_i^{(1)}\right)k_4^{(2)} + \left(k_i^{(2)} + 2p_i\right)k_4^{(1)}\right] - \\ -\varepsilon_0\left(\left(k_i^{(2)} - 2p_i\right)k_4^{(1)} + \left(k_i^{(1)} + 2p_i\right)k_4^{(2)}\right)\left(H_2 + H_3\right) \end{array}\right\}$$

$$F_{ij4}\approx F_{ij4}^{\ stat.}+$$

$$\mathrm{Re}\left\{\begin{array}{l}\varepsilon_0\delta_{ij}\left(H_2\varepsilon_1k_4^{(1)}+H_3\varepsilon_2k_4^{(2)}\right)+H_1\left[\begin{array}{l}\left(\mathbf{pk}^{(2)}\delta_{ij}+2p_ip_j-p_ik_j^{(2)}-p_jk_i^{(2)}\right)k_4^{(1)}+\\+\left(\mathbf{pk}^{(1)}\delta_{ij}+2p_ip_j-p_ik_j^{(1)}-p_jk_i^{(1)}\right)k_4^{(2)}\end{array}\right]+\\H_2\left[\begin{array}{l}\left(\left(\mathbf{p}\left(\mathbf{2k^{(1)}}+\mathbf{k^{(2)}}\right)+\left(\mathbf{k^{(1)}}\right)^2\right)\delta_{ij}+2p_ip_j+p_ik_j^{(2)}+p_jk_i^{(2)}\right)k_4^{(1)}-\\-\left(\left(\mathbf{pk^{(1)}}+\left(\mathbf{k^{(1)}}\right)^2\right)\delta_{ij}+2p_ip_j+p_ik_j^{(1)}+p_jk_i^{(1)}\right)k_4^{(2)}\end{array}\right]+\\H_3\left[\begin{array}{l}\left(\left(-\mathbf{pk}^{(2)}+\left(\mathbf{k}^{(2)}\right)^2\right)\delta_{ij}+2p_ip_j-p_ik_j^{(2)}-p_jk_i^{(2)}\right)k_4^{(1)}+\\+\left(\left(\mathbf{p}\left(\mathbf{2k}^{(2)}+\mathbf{k}^{(1)}\right)+\left(\mathbf{k}^{(2)}\right)^2\right)\delta_{ij}+2p_ip_j+p_ik_j^{(1)}+p_jk_i^{(1)}\right)k_4^{(2)}\end{array}\right]\end{array}\right\}$$

$$F_{ijl}\approx\mathrm{Im}\left\{\begin{array}{l}H_1\varepsilon_0\left[\begin{array}{l}\left(k_l^{(2)}\delta_{ij}+k_j^{(2)}\delta_{il}-k_i^{(2)}\delta_{jl}-2p_j\delta_{il}\right)k_4^{(1)}+\\+\left(k_l^{(1)}\delta_{ij}-k_j^{(1)}\delta_{il}+k_i^{(1)}\delta_{jl}+2p_i\delta_{jl}\right)k_4^{(2)}\end{array}\right]\\H_2\varepsilon_1\left[\begin{array}{l}\left(\begin{array}{l}k_i^{(2)}\delta_{jl}-k_l^{(2)}\delta_{ij}-k_j^{(2)}\delta_{il}+\left(k_i^{(1)}+2p_i\right)\\\times\delta_{jl}+\left(k_l^{(1)}+p_l\right)\delta_{ij}-\left(k_j^{(1)}+p_j\right)\delta_{il}\end{array}\right)k_4^{(1)}+\\+\left(k_l^{(1)}\delta_{ij}-k_j^{(1)}\delta_{il}+k_i^{(1)}\delta_{jl}+2p_i\delta_{jl}\right)k_4^{(2)}\end{array}\right]+\\H_3\varepsilon_2\left[\begin{array}{l}\left(\begin{array}{l}k_i^{(1)}\delta_{jl}-k_l^{(1)}\delta_{ij}-k_j^{(1)}\delta_{il}+\left(k_i^{(2)}-2p_i\right)\\\times\delta_{jl}+\left(k_l^{(2)}-p_l\right)\delta_{ij}-\left(k_j^{(2)}-p_j\right)\delta_{il}\end{array}\right)k_4^{(2)}+\\+\left(k_l^{(2)}\delta_{ij}-k_j^{(2)}\delta_{il}+k_i^{(2)}\delta_{jl}-2p_i\delta_{jl}\right)k_4^{(1)}\end{array}\right]-\end{array}\right\}$$

Here we give the general view of the tensor function for the case of using the perturbation theory

$$\Pi_{\mu\nu\gamma}(k,k^{'},k^{''})=\Pi_{\mu\nu\gamma}{}^{m}(k,k^{'},k^{''})+\frac{\delta(k+k^{'}+k^{''})e^{3}}{(2\pi)^{3}}\times$$

$$\times\sum_{f}\int dp^{4}\begin{pmatrix}(\gamma_{\mu}G(p+k)\gamma_{\nu}G(p-k^{b}-k^{'})\gamma_{\gamma}G(p-k^{b})\gamma_{f}G(p)+\\ +\gamma_{\mu}G(p)\gamma_{\nu}G(p+k^{'})\gamma_{\gamma}G(p+k^{b}-k^{'})\gamma_{f}G(p+k^{b})+\\ +\gamma_{\mu}G(p+k-k^{b})\gamma_{f}G(p-k^{b}-k^{'})\gamma_{\nu}G(p-k^{'})\gamma_{\gamma}G(p)\\ +\gamma_{\mu}G(p-k+k^{b})\gamma_{f}G(p+k^{b}+k^{'})\gamma_{\nu}G(p+k^{'})\gamma_{\gamma}G(p)\\ +\gamma_{\mu}G(p+k)\gamma_{\nu}G(p-k^{b}-k^{'})\gamma_{f}G(p-k^{'})\gamma_{\gamma}G(p)\\ +\gamma_{\mu}G(p-k)\gamma_{\nu}G(p+k^{b}+k^{'})\gamma_{f}G(p+k^{'})\gamma_{\gamma}G(p))\end{pmatrix}$$

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