UDC 533.72

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ON THE KINETIC EQUATION FOR A MANY-BODY DISSIPATIVE RANDOMLY DRIVEN SYSTEM

The present paper is based on the Bogolyubov-Born-Green-Kirkwood-Yvon hierarchy for many-body one-component dissipative systems in an external stochastic field. This hierarchy was obtained recently by Yu. V. Slyusarenko, O. Yu. Sliusarenko, and A. V. Chechkin. These authors obtained also an analog of the Vlasov kinetic equation for the above-mentioned systems. Namely, they obtained a kinetic equation up to the first order in small interaction, dissipation and external stochastic field. On the basis of their kinetic equation they showed that the momentum of the system is conserved and obtained the kinetic energy of the system as a function of time for some simple cases. They also discussed under which conditions the Maxwellian distribution is an equilibrium one. This paper is devoted to obtaining the analog of the Landau collision integral for such type of systems. The kinetic equation is obtained up to the second order in small interaction, dissipation and external stochastic field. On the basis of this equation it is shown that the momentum of the Maxwellian gas is conserved in some simple cases.

Keywords: Landau–Vlasov kinetic equation, small interaction, small dissipation, small stochastic external field, momentum conservation.

Робота базується на ланцюжку Боголюбова–Борна–Гріна–Кірквуда–Івона для багаточастинкових дисипативних систем у зовнішньому стохастичному полі. Цей ланцюжок нещодавно отримали Ю. В. Слюсаренко, О. Ю. Слюсаренко та О. В. Чечкін. Ці автори також отримали аналог кінетичного рівняння Власова для вищеназваних систем. А саме, ними отримано кінетичне рівняння до членів першого порядку за малими взаємодією, дисипацією та зовнішнім стохастичним полем. На основі свого рівняння для деяких простих випадків вони показали, що імпульс системи зберігається, й отримали кінетичну енергію як функцію часу, а також дослідили, за яких умов рівноважним розподілом є розподіл Максвелла. Дана робота присвячена отриманню аналога інтеграла зіткнень Ландау для таких систем. Знайдено кінетичне рівняння до членів другого порядку за малими взаємодією, дисипацією та зовнішнім стохастичним полем. На основі одержаного рівняння показано, що імпульс максвелівського газу зберігається для деяких простих випадків.

Ключові слова: кінетичне рівняння Ландау–Власова, мала взаємодія, мала дисипація, мале зовнішнє стохастичне поле, збереження імпульсу.

Работа базируется на цепочке Боголюбова–Борна–Грина–Кирквуда–Ивона для многочастичных однокомпонентных диссипативных систем во внешнем стохастическом поле. Эта цепочка недавно была получена Ю. В. Слюсаренко, А. Ю. Слюсаренко и А. В. Чечкиным. Эти авторы также получили аналог кинетического уравнения Власова для названных систем. А именно, ими получено кинетическое уравнение до членов первого порядка малости по малым взаимодействию, диссипации и внешнему стохастическому полю. На основе своего уравнения для некоторых простых случаев они показали, что импульс системы сохраняется, и получили кинетическую энергию как функцию времени, а также исследовали, при каких условиях распределение Максвелла является равновесным. Данная работа посвящена получению аналога интеграла столкновений Ландау для таких систем. Получено кинетическое уравнение до членов второго порядка по малым взаимодействию, диссипации и внешнему стохастическому полю. На основе выведенного уравнения показано, что импульс максвелловского газа сохраняется для некоторых простых случаев.

Ключевые слова: кинетическое уравнение Ландау–Власова, малое взаимодействие, малая диссипация, малое внешнее стохастическое поле, сохранение импульса.

1. Introduction

Recently Yu. V. Slyusarenko, O. Yu. Sliusarenko and A. V. Chechkin obtained [1] a Bogolyubov–Born–Green–Kirkwood–Yvon hierarchy for many-body one-component dissipative systems in an external stochastic field. The investigation of such systems is important for modern statistical physics, see [1] and the references therein. On the basis of the obtained hierarchy and the Bogolyubov reduced description method in the case of weak interaction, weak dissipation and weak external stochastic field they derived an analog of the Vlasov kinetic equation for the above-mentioned systems. In other words, they obtained a kinetic equation up to the first order in small interaction, small dissipation and small external stochastic field. On the basis of their kinetic equation in some simple cases they showed that the momentum and particle density are conserved, and they obtained the kinetic energy as a function of time. They also investigated under which conditions the Maxwellian distribution is an equilibrium one.

As known [2], in the case of a non-dissipative system without any external field the well-known result is a kinetic equation up to the second order in small interaction. In contrast to the Vlasov kinetic equation, it contains a term of the second order in small interaction. Its local part is called the Landau collision integral. The kinetic equation with the Landau collision integral and the Vlasov self-consistent field is widely used in modern statistical physics, in particular in the physics of plasma, see, for example, [3,4]. The Landau collision integral is a local collision integral – it is of the leading order in small gradients of the one-particle component distribution function. It is important to obtain an analog of the Landau collision integral for dissipative systems in an external stochastic field. In other words, an important problem is to derive a kinetic equation up to the second order in small interaction, small dissipation and small external stochastic field. The importance of this problem is also stressed in [1]. Special attention is paid to local second-order terms.

An analog of the Landau collision integral for the above-mentioned systems is obtained. The properties of a spatially uniform Maxwellian gas are investigated on the basis of the obtained analog. It is shown that in some simple cases the momentum of a spatially uniform Maxwellian gas is conserved.

The paper is organised as follows. In the section 2 the basic equations of the theory are given. The section 3 is devoted to obtaining a collision integral of the second order in small interaction, small dissipation and small external stochastic field. The section 4 is devoted to the investigation of the momentum conservation of a Maxwellian gas in some simple cases, and in the section 5 the conclusions are given.

2. Basic equations of the theory

The work is based on the the Bogolyubov–Born–Green–Kirkwood–Yvon hierarchy for many-body one-component dissipative systems in an external stochastic field which was obtained in [1]:

$$\begin{aligned}\frac{\partial f_S}{\partial t} = &-\sum_{1\le\alpha\le S}\frac{\mathbf{p}_\alpha}{m}\frac{\partial f_S}{\partial \mathbf{x}_\alpha} - \sum_{1\le\alpha<\beta\le S}\left(\frac{\partial}{\partial \mathbf{p}_\alpha}-\frac{\partial}{\partial \mathbf{p}_\beta}\right) f_S\mathbf{F}_{\alpha\beta} - \sum_{1\le\alpha\le S}\frac{\partial}{\partial \mathbf{p}_\alpha}\int d\chi_{S+1} f_{S+1}\mathbf{F}_{\alpha,S+1} - \\ &-\sum_{r=2}^{\infty}\frac{(-1)^{r-1}}{2^{r-1}(r-1)!}\sum_{1\le\alpha_1\le S}\cdots\sum_{1\le\alpha_r\le S} y_{i_1i_2\ldots i_r}\left(\mathbf{x}_{\alpha_1},\mathbf{x}_{\alpha_2},\ldots,\mathbf{x}_{\alpha_r}\right)\frac{\partial^r f_S}{\partial p_{\alpha_1 i_1}\ldots\partial p_{\alpha_r i_r}}\end{aligned} \qquad (1)$$

where $f_S = f_S(\chi_1,\ldots,\chi_s,t)$ is the S-particle distribution function, $\chi_i \equiv \{\mathbf{x}_i,\mathbf{p}_i\}$, $y_{i_1i_2\ldots i_r}$ are the correlation functions of the external stochastic field (see [1]), and

$$\mathbf{F}_{\alpha\beta} \equiv -\partial V_{\alpha\beta}/\partial \mathbf{x}_\alpha - \partial R_{\alpha\beta}/\partial \mathbf{p}_\alpha , \ R = \sum\nolimits_{1\le\alpha<\beta\le N} R_{\alpha\beta} \tag{2}$$

where $V_{\alpha\beta} \equiv V\left(\left|\mathbf{x}_\alpha - \mathbf{x}_\beta\right|\right)$ is the interaction potential between the particles in the system, R is the dissipation function, $R_{\alpha\beta} \equiv R\left(\mathbf{x}_\alpha - \mathbf{x}_\beta, \mathbf{p}_\alpha - \mathbf{p}_\beta\right)$. Here m is the mass of one particle in the system and N is the total number of particles in the system. Note that we consider the case where the average value of the external stochastic field is equal to zero.

The further investigation is based on the principle of spatial weakening of correlations and on the Bogolyubov functional hypothesis [1,2]:

$$f_S\left(\chi_1,\chi_2,...,\chi_S,t\right) \xrightarrow[t\gg\tau_0]{} f_S\left(\chi_1,\chi_2,...,\chi_S,f_1(t)\right) \tag{3}$$

where τ_0 is some characteristic time which is the time of beginning of the kinetic stage of evolution. The kinetic equation is a time equation for the one-particle distribution function. On the basis of (1), (3) it has the form

$$\frac{\partial f_1(\chi_1)}{\partial t} = -\frac{\mathbf{p}_1}{m}\frac{\partial f_1(\chi_1)}{\partial \mathbf{x}_1} - \sum_{r=2}^{\infty}\frac{(-1)^{r-1}}{2^{r-1}(r-1)!} y_{i_1 i_2 \dots i_r}\left(\mathbf{x}_1,\mathbf{x}_1,...,\mathbf{x}_1\right)\frac{\partial^r f_1(\chi_1)}{\partial p_{1i_1}\dots\partial p_{1i_r}} + L(\chi_1, f_1) \tag{4}$$

where

$$L(\chi_1, f_1) = -\frac{\partial}{\partial \mathbf{p}_1}\int d\chi_2 f_2\left(\chi_1,\chi_2,f_1\right)\mathbf{F}_{12}\,. \tag{5}$$

It is shown [1] that

$$\begin{gathered}
f_S\left(\chi_1,...,\chi_S\right) = \prod_{i=1}^{S} f_1\left(\chi_i\right) + \int_{-\infty}^{0} d\tau e^{\tau\Lambda_S} K_S\left(\chi_1,...,\chi_S, e^{-\tau\Lambda_1} f_1\right), \\
K_S\left(f_1\right) = -\int d\chi \frac{\delta f_S\left(f_1\right)}{\delta f_1(\chi)} L(\chi, f_1) - \sum_{1\le\alpha<\beta\le S}\left(\frac{\partial}{\partial \mathbf{p}_\alpha} - \frac{\partial}{\partial \mathbf{p}_\beta}\right) f_S \mathbf{F}_{\alpha\beta} + \\
+\sum_{r=2}^{\infty}\frac{(-1)^{r-1}}{2^{r-1}(r-1)!}\int d\chi \frac{\delta f_S\left(\chi_1,\chi_2,...,\chi_S,f_1\right)}{\delta f_1(\chi)} y_{i_1 i_2\dots i_r}\left(\mathbf{x},\mathbf{x},...,\mathbf{x}\right)\frac{\partial^r f_1}{\partial p_{i_1}\dots\partial p_{i_r}} - \\
-\sum_{r=2}^{\infty}\frac{(-1)^{r-1}}{2^{r-1}(r-1)!}\sum_{1\le\alpha_1\le S}\dots\sum_{1\le\alpha_r\le S} y_{i_1 i_2\dots i_r}\left(\mathbf{x}_{\alpha_1},\mathbf{x}_{\alpha_2},...,\mathbf{x}_{\alpha_r}\right)\frac{\partial^r f_S}{\partial p_{\alpha_1 i_1}\dots\partial p_{\alpha_r i_r}} - \\
-\sum_{1\le\alpha\le S}\frac{\partial}{\partial \mathbf{p}_\alpha}\int d\chi_{S+1} f_{S+1}\mathbf{F}_{\alpha,S+1}, \\
e^{\tau\Lambda_S} f\left(\mathbf{x}_j,\mathbf{p}_j\right) \equiv f\left(\mathbf{x}_j + \tau\mathbf{p}_j/m, \mathbf{p}_j\right), \ j = 1,2,...,S\,.
\end{gathered} \tag{6}$$

The interaction potential V, the dissipation function R and the correlation functions $y_{i_1 i_2 \dots i_r}$ are assumed to be small, and for simplicity they are estimated by one small parameter Φ. Obviously,

$$f_2^{(0)}\left(\chi_1,\chi_2\right) = f_1\left(\chi_1\right) f_1\left(\chi_2\right), \tag{7}$$

here and in what follows, the superscript in parentheses denotes the order in Φ. So,

$$L^{(1)}(\chi_1, f_1) = \frac{\partial}{\partial \mathbf{p}_1} f_1(\chi_1) \int d\chi_2 f_1(\chi_2) \left(\frac{\partial V_{12}}{\partial \mathbf{x}_1} + \frac{\partial R_{12}}{\partial \mathbf{p}_1} \right). \tag{8}$$

If we substitute (8) into (4) instead of $L(\chi_1, f_1)$, we will obtain the Slyusarenko–Sliusarenko–Chechkin kinetic equation which is discussed in detail in [1].

3. Derivation of the second-order collision integral

The aim of this paper is to obtain $L^{(2)}(\chi_1, f_1)$. Obviously,

$$f_2^{(1)}(\chi_1, \chi_2) = \int_{-\infty}^{0} d\tau e^{\tau\Lambda_2} K_2\left(\chi_1, \chi_2, e^{-\tau\Lambda_1} f_1\right). \tag{9}$$

After a lengthy calculation we obtain

$$\begin{gathered}
f_2^{(1)} = f_{2V}^{(1)} + f_{2R}^{(1)} + f_{2y}^{(1)}, \\
f_{2V}^{(1)} = \int_{-\infty}^{0} d\tau \left(\frac{\partial f_1(\chi_1)}{\partial \mathbf{p}_1} f_1(\chi_2) - f_1(\chi_1) \frac{\partial f_1(\chi_2)}{\partial \mathbf{p}_2} \right) \frac{\partial V_{12\tau}}{\partial \mathbf{x}_1} + \\
+ \int_{-\infty}^{0} d\tau \frac{\tau}{m} \left\{ f_1(\chi_1) \frac{\partial f_1(\chi_2)}{\partial \mathbf{x}_2} - \frac{\partial f_1(\chi_1)}{\partial \mathbf{x}_1} f_1(\chi_2) \right\} \frac{\partial V_{12\tau}}{\partial \mathbf{x}_1}, \\
V_{12\tau} \equiv V\left(\left| \mathbf{x}_1 - \mathbf{x}_2 + \tau(\mathbf{p}_1 - \mathbf{p}_2)/m \right| \right), \\
f_{2R}^{(1)} = \int_{-\infty}^{0} d\tau \left(\frac{\partial f_1(\chi_1)}{\partial \mathbf{p}_1} f_1(\chi_2) - f_1(\chi_1) \frac{\partial f_1(\chi_2)}{\partial \mathbf{p}_2} \right) \frac{\partial R_{12\tau}}{\partial \mathbf{p}_1} + \\
+ \int_{-\infty}^{0} d\tau \frac{\tau}{m} \left\{ f_1(\chi_1) \frac{\partial f_1(\chi_2)}{\partial \mathbf{x}_2} - \frac{\partial f_1(\chi_1)}{\partial \mathbf{x}_1} f_1(\chi_2) \right\} \frac{\partial R_{12\tau}}{\partial \mathbf{p}_1} + \\
+ \int_{-\infty}^{0} d\tau f_1(\chi_1) f_1(\chi_2) \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2} \right) \frac{\partial R_{12\tau}}{\partial \mathbf{p}_1}, \\
R_{12\tau} \equiv R\left(\mathbf{x}_1 - \mathbf{x}_2 + \tau(\mathbf{p}_1 - \mathbf{p}_2)/m, \mathbf{p}_1 - \mathbf{p}_2 \right), \\
f_{2y}^{(1)} = \sum_{r=2}^{\infty} \frac{(-1)^r}{2^{r-1}(r-1)!} \int_{-\infty}^{0} d\tau \sum_{1 \le \alpha_1 \le 2} \dots \sum_{1 \le \alpha_r \le 2} y_{i_1 i_2 \dots i_r \tau} \left. \frac{\partial^r f_1(\chi_1) f_1(\chi_2)}{\partial p_{\alpha_1 i_1} \dots \partial p_{\alpha_r i_r}} \right|_{\substack{\text{the } \alpha_i\text{'s are} \\ \text{not all equal} \\ \text{to one another}}} + f_{2y}'^{(1)}, \\
y_{i_1 i_2 \dots i_r \tau} \equiv y_{i_1 i_2 \dots i_r}\left(\mathbf{x}_{\alpha_1} + \tau \mathbf{p}_{\alpha_1}/m, \mathbf{x}_{\alpha_2} + \tau \mathbf{p}_{\alpha_2}/m, \dots, \mathbf{x}_{\alpha_r} + \tau \mathbf{p}_{\alpha_r}/m \right)
\end{gathered} \tag{10}$$

where $f_{2y}'^{(1)}$ is a cumbersome expression which contains the spatial derivatives of $f_1(\chi_1)$ and $f_1(\chi_2)$.

In this paper special attention is paid to the derivation of the local collision integral. It is the collision integral of the leading order in small spatial gradients of the distribution function. Local collision integrals play a huge role in modern statistical physics, usually hydrodynamics up to the first order in small gradients and kinetic coefficients are investigated on the basis of local collision integrals, see [5, 6]. Also note that the well-known Landau collision integral is a local one [2]. In the leading order in gradients, expressions (10) take the form

$$f_{2V}^{(1L)} = \int_{-\infty}^{0} d\tau \left(\frac{\partial f_{p1}}{\partial \mathbf{p}_1} f_{p2} - f_{p1} \frac{\partial f_{p2}}{\partial \mathbf{p}_2} \right) \frac{\partial V_{12\tau}}{\partial \mathbf{x}_1},$$

$$f_{2R}^{(1L)} = \int_{-\infty}^{0} d\tau \left(\frac{\partial f_{p1}}{\partial \mathbf{p}_1} f_{p2} - f_{p1} \frac{\partial f_{p2}}{\partial \mathbf{p}_2} \right) \frac{\partial R_{12\tau}}{\partial \mathbf{p}_1} + \int_{-\infty}^{0} d\tau f_{p1} f_{p2} \left(\frac{\partial}{\partial \mathbf{p}_1} - \frac{\partial}{\partial \mathbf{p}_2} \right) \frac{\partial R_{12\tau}}{\partial \mathbf{p}_1}, \tag{11}$$

$$f_{2y}^{(1L)} = \sum_{r=2}^{\infty} \frac{(-1)^r}{2^{r-1}(r-1)!} \int_{-\infty}^{0} d\tau \sum_{1 \le \alpha_1 \le 2} \dots \sum_{1 \le \alpha_r \le 2} y_{i_1 i_2 \dots i_r \tau} \left. \frac{\partial^r f_{p1} f_{p2}}{\partial p_{\alpha_1 i_1} \dots \partial p_{\alpha_r i_r}} \right|_{\text{the } \alpha_i\text{'s are not all equal to one another}},$$

$$f_{p1} \equiv f_1(\mathbf{x}_1, \mathbf{p}_1), \; f_{p2} \equiv f_2(\mathbf{x}_1, \mathbf{p}_2),$$

here and in what follows the superscript L denotes "local". So, the local collision integral is given by the expressions

$$L^{(2L)} = L_{VV}^{(2L)} + L_{RV}^{(2L)} + L_{yV}^{(2L)} + L_{VR}^{(2L)} + L_{RR}^{(2L)} + L_{yR}^{(2L)},$$

$$L_{VV}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2V}^{(1L)} \frac{\partial V_{12}}{\partial \mathbf{x}_1}, \quad L_{RV}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2R}^{(1L)} \frac{\partial V_{12}}{\partial \mathbf{x}_1},$$

$$L_{yV}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2y}^{(1L)} \frac{\partial V_{12}}{\partial \mathbf{x}_1}, \quad L_{VR}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2V}^{(1L)} \frac{\partial R_{12}}{\partial \mathbf{p}_1}, \tag{12}$$

$$L_{RR}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2R}^{(1L)} \frac{\partial R_{12}}{\partial \mathbf{p}_1}, \quad L_{yR}^{(2L)} = \frac{\partial}{\partial \mathbf{p}_1} \int d\chi_2 f_{2y}^{(1L)} \frac{\partial R_{12}}{\partial \mathbf{p}_1},$$

The functions $f_{2V}^{(1L)}$, $f_{2R}^{(1L)}$ and $f_{2y}^{(1L)}$ are given in (11). By straightforward calculation it can be shown that $L_{VV}^{(2L)}$ coincides with the well-known Landau collision integral [2], so in the well-known case where there is no dissipation and no external field, our result coincides with the result known in the literature. This fact justifies the results (11), (12).

As known, the particle and momentum densities of the system are introduced by standard definitions:

$$n \equiv \int d^3 p f_p, \quad \pi_l \equiv \int d^3 p p_l f_p. \tag{13}$$

Obviously, L has no effect on the time equation for n because the integral of the divergence is equal to zero. In what follows, the effect of $L^{(2L)}$ on the time equation of the momentum of a Maxwellian gas is discussed.

4. Conservation of the momentum of a spatially homogenous Maxwellian gas in some simple cases

This section is devoted to the calculation of the effect of $L^{(2L)}$ on the time equation for the momentum density of a spatially uniform Maxwellian gas. On the basis of the Slyusarenko–Sliusarenko–Chechkin kinetic equation it is shown [1] that the momentum of any spatially-uniform one-component dissipative system in an external stochastic field is conserved in the framework of a simple model where

$$R_{12} = \gamma(\mathbf{x}_1 - \mathbf{x}_2) \cdot (\mathbf{p}_1 - \mathbf{p}_2)^2 / 2, \quad \gamma(\mathbf{x}_1 - \mathbf{x}_2) > 0. \tag{14}$$

As also mentioned in [1], in a spatially uniform system the correlation functions $y_{i_1 i_2 \dots i_r}$ depend only on the coordinate differences:

$$y_{i_1 i_2 \dots i_r}\left(\mathbf{x}_{\alpha_1}, \mathbf{x}_{\alpha_2}, \dots, \mathbf{x}_{\alpha_r}\right) = y_{i_1 i_2 \dots i_r}\left(\mathbf{x}_{\alpha_i} - \mathbf{x}_{\alpha_j}\right), \quad 1 \le i < j \le r . \tag{15}$$

On the basis of (14) and (15) Slyusarenko, Sliusarenko and Chechkin obtained an explicit time dependence of the kinetic energy of an arbitrary spatially-uniform one-component dissipative system in an external stochastic field. They also investigated the conditions under which the Maxwellian distribution function is an equilibrium one.

As to the investigation of the effect of $L^{(2L)}$ on the conservation laws, the situation is much more complicated. So, in order to deal with it, we consider further simplifications of (14) and (15). For simplicity, we investigate the case where

$$\begin{gathered} R_{12} = \gamma\left(\left|\mathbf{x}_1 - \mathbf{x}_2\right|\right) \cdot \left(\mathbf{p}_1 - \mathbf{p}_2\right)^2 / 2, \quad \gamma\left(\left|\mathbf{x}_1 - \mathbf{x}_2\right|\right) > 0; \\ y_{i_1 i_2 \dots i_r}\left(\mathbf{x}_{\alpha_1}, \mathbf{x}_{\alpha_2}, \dots, \mathbf{x}_{\alpha_r}\right) = y_{i_1 i_2 \dots i_r}\left(\left|\mathbf{x}_{\alpha_i} - \mathbf{x}_{\alpha_j}\right|\right), \quad 1 \le i < j \le r . \end{gathered} \tag{16}$$

On the basis of (13), (4) and (12) we obtain

$$\left(\partial_t \pi_l\right)^{(2L)} = \left(\partial_t \pi_l\right)_{VV}^{(2L)} + \left(\partial_t \pi_l\right)_{RV}^{(2L)} + \left(\partial_t \pi_l\right)_{yV}^{(2L)} + \left(\partial_t \pi_l\right)_{VR}^{(2L)} + \left(\partial_t \pi_l\right)_{RR}^{(2L)} + \left(\partial_t \pi_l\right)_{yR}^{(2L)} \tag{17}$$

where

$$\begin{gathered} \left(\partial_t \pi_l\right)_{VV}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2V}^{(1)} \, \partial V_{12} / \partial x_{1l}, \ \left(\partial_t \pi_l\right)_{RV}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2R}^{(1)} \, \partial V_{12} / \partial x_{1l}, \\ \left(\partial_t \pi_l\right)_{yV}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2y}^{(1)} \, \partial V_{12} / \partial x_{1l}, \ \left(\partial_t \pi_l\right)_{VR}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2V}^{(1)} \, \partial R_{12} / \partial p_{1l}, \\ \left(\partial_t \pi_l\right)_{RR}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2R}^{(1)} \, \partial R_{12} / \partial p_{1l}, \ \left(\partial_t \pi_l\right)_{yR}^{(2L)} = -\int d^3 p_1 d\chi_2 f_{2y}^{(1)} \, \partial R_{12} / \partial p_{1l}. \end{gathered} \tag{18}$$

The well-known result [2] gives $\left(\partial_t \pi_l\right)_{VV}^{(2L)} = 0$. Using the Fourier transforms and the property of δ-function

$$\begin{gathered} \gamma\left(\left|\mathbf{x}_1 - \mathbf{x}_2 + \tau\left(\mathbf{p}_1 - \mathbf{p}_2\right)/m\right|\right) = (2\pi)^{-3} \int d^3 k e^{i\mathbf{k}\left(\mathbf{x}_1 - \mathbf{x}_2 + \tau\left(\mathbf{p}_1 - \mathbf{p}_2\right)/m\right)} \gamma(k), \ k = |\mathbf{k}|, \\ V_{12} = V\left(\left|\mathbf{x}_1 - \mathbf{x}_2\right|\right) = (2\pi)^{-3} \int d^3 k e^{i\mathbf{k}\left(\mathbf{x}_1 - \mathbf{x}_2\right)} V(k), \\ \int d^3 x_2 e^{i(\mathbf{k}' + \mathbf{k})(\mathbf{x}_1 - \mathbf{x}_2)} = (2\pi)^3 \, \delta(\mathbf{k}' + \mathbf{k}), \end{gathered} \tag{19}$$

we obtain the following expression for $\left(\partial_t \pi_l\right)_{VR}^{(2L)}$:

$$\begin{gathered} \left(\partial_t \pi_l\right)_{VR}^{(2L)} = -(2\pi)^{-3} \, i \int d^3 p_1 d^3 p_2 \left(\frac{\partial f_{p1}}{\partial p_{1n}} f_{p2} - f_{p1} \frac{\partial f_{p2}}{\partial p_{2n}}\right) \left(p_{1l} - p_{2l}\right) \times \\ \times \int_{-\infty}^{0} d\tau \int d^3 k e^{i\tau \mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)/m} \gamma(k) V(k) k_n . \end{gathered} \tag{20}$$

Substituting the Maxwellian distribution function

$$f_p = n(2\pi m T)^{-3/2} \, e^{-(\mathbf{p} - m\mathbf{v})^2 / 2mT} \tag{21}$$

into (20), we obtain

$$\left(\partial_t \pi_l\right)_{VR}^{(2L)} = \frac{i}{(2\pi)^3} \frac{n^2}{(2\pi mT)^3} \int d^3 p_1 d^3 p_2 e^{-\frac{(\mathbf{p}_1 - m\mathbf{v})^2 + (\mathbf{p}_2 - m\mathbf{v})^2}{2mT}} \frac{p_{1n} - p_{2n}}{mT} \left(p_{1l} - p_{2l}\right) \times$$
$$\times \int_{-\infty}^{0} d\tau \int d^3 k e^{i\tau \mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)/m} \gamma(k) V(k) k_n . \tag{22}$$

Introducing the following variables

$$\mathbf{p}_1' = \mathbf{p}_1 - m\mathbf{v},\ \mathbf{p}_2' = \mathbf{p}_2 - m\mathbf{v},\ \mathbf{y} = \mathbf{p}_1' + \mathbf{p}_2',\ \mathbf{z} = \mathbf{p}_1' - \mathbf{p}_2' , \tag{23}$$

we obtain

$$\left(\partial_t \pi_l\right)_{VR}^{(2L)} = i (4\pi)^{-3} n^2 \pi^{-3/2} (mT)^{-5/2} \int d^3 k \gamma(k) V(k) \int_{-\infty}^{0} d\tau \int d^3 z z_l \mathbf{kz} e^{i\mathbf{kz}\tau/m} e^{-z^2/4mT} . \tag{24}$$

Due to the rotational invariance $\int_{-\infty}^{0} d\tau \int d^3 z z_l \mathbf{kz} e^{i\tau \mathbf{kz}/m} e^{-z^2/4mT} = k_l \Theta(k)$ where $\Theta(k)$ is some function of $|\mathbf{k}|$. So, $\left(\partial_t \pi_l\right)_{VR}^{(2L)} = 0$ because the integrand is an odd function of $\mathbf{k}$. Similarly, it can be shown that $\left(\partial_t \pi_l\right)_{RV}^{(2L)} = 0$, $\left(\partial_t \pi_l\right)_{RR}^{(2L)} = 0$.

Using the Fourier transform

$$y_{i_1 i_2 \dots i_r}\left(\left|\mathbf{x}_1 - \mathbf{x}_2 + \tau(\mathbf{p}_1 - \mathbf{p}_2)/m\right|\right) = (2\pi)^{-3} \int d^3 k e^{i\mathbf{k}(\mathbf{x}_1 - \mathbf{x}_2 + \tau(\mathbf{p}_1 - \mathbf{p}_2)/m)} y_{i_1 i_2 \dots i_r}(k) \tag{25}$$

we obtain

$$\left(\partial_t \pi_l\right)_{yV}^{(2L)} = \frac{-i}{(2\pi)^3} \sum_{r=2}^{\infty} \frac{(-1)^{r-1}}{2^{r-1}(r-1)!} \sum_{1 \le \alpha_1 \le 2} \dots \sum_{1 \le \alpha_r \le 2} \int d^3 p_1 d^3 p_2 \frac{\partial^r f_{p1} f_{p2}}{\partial p_{\alpha_1 i_1} \dots \partial p_{\alpha_r i_r}} \times$$
$$\times \int_{-\infty}^{0} d\tau \int d^3 k e^{i\mathbf{k}\tau(\mathbf{p}_1 - \mathbf{p}_2)/m} k_l \gamma(k) y_{i_1 i_2 \dots i_r}(k) \Big|_{\text{the } \alpha_i\text{'s are not all equal to one another}} . \tag{26}$$

Let us consider the combination where $\alpha_1 = \alpha_2 = \dots = \alpha_s = 1$, $\alpha_{s+1} = \alpha_{s+2} = \dots = \alpha_r = 2$. Then after r integrations by parts we obtain

$$\int d^3 p_1 d^3 p_2 \frac{\partial^s f_{p1}}{\partial p_{1i_1} \dots \partial p_{1i_s}} \frac{\partial^{r-s} f_{p2}}{\partial p_{2i_{s+1}} \dots \partial p_{2i_r}} \int_{-\infty}^{0} d\tau \int d^3 k e^{i\mathbf{k}\frac{\tau}{m}(\mathbf{p}_1 - \mathbf{p}_2)} k_l V(k) y_{i_1 i_2 \dots i_r}(k) =$$
$$= (-1)^s \int d^3 p_2 d^3 p_1 f_{p1} f_{p2} \int_{-\infty}^{0} d\tau (i\tau/m)^r \int d^3 k k_{i_1} \dots k_{i_r} k_q e^{i\mathbf{k}(\mathbf{p}_1 - \mathbf{p}_2)\tau/m} V(k) y_{i_1 i_2 \dots i_r}(k) . \tag{27}$$

Obviously, any combination with s of the α_i's equal to 1 and $r - s$ of the α_i's equal to 2 gives the same answer (27). There are $C_r^s = r!/\left((r-s)!\, s!\right)$ such combinations. So, using (21), (23) and (27), we have

$$\left(\partial_t \pi_l\right)_{yV}^{(2L)} = -i (4\pi)^{-3} (\pi mT)^{-3/2} n^2 \sum_{r=3,5,7,\dots} \frac{(-1)^{r-1}}{2^{r-1}(r-1)!} \times$$
$$\times \sum_{s=1}^{r-1} (-1)^s C_r^s \int d^3 k k_{i_1} \dots k_{i_r} k_q V(k) y_{i_1 i_2 \dots i_r}(k) \int_{-\infty}^{0} d\tau (i\tau/m)^r \int d^3 z e^{i\mathbf{kz}\tau/m} e^{-z^2/4mT} , \tag{28}$$

here $r = 3, 5, 7, \dots$ because for an even r due to the rotational invariance we have an integrand which is odd in $\mathbf{k}$. But for an odd r the following identity is valid:

$\sum_{s=1}^{r-1}(-1)^s C_r^s = 0$, so $(\partial_t \pi_l)_{yV}^{(2L)} = 0$. Similarly, $(\partial_t \pi_l)_{yR}^{(2L)} = 0$. So, the momentum of the Maxwellian gas is conserved under the conditions (16). Also note that the same calculations are valid if the correlation functions $y_{i_1 i_2 \ldots i_r}$ and the function γ are even functions of the coordinate differences and such severe restrictions as (16) are not necessary. So, if the correlation functions $y_{i_1 i_2 \ldots i_r}$ and the function γ are even functions of the coordinate differences, the momentum of a Maxwellian gas is conserved too.

5. Conclusions

An analog of the Landau collision integral is obtained for many-body one-component dissipative systems in an external stochastic field. In other words, the kinetic equation is obtained up to the second order in small interaction, small dissipation and small external stochastic field. This result is an extension of the Slyusarenko–Sliusarenko–Chechkin kinetic equation, which was obtained [1] up to the first order in the above-mentioned small parameters.

As known, the local collision integral plays a very important role in modern statistical physics. For example, usually it is the basis of the hydrodynamics investigation up to the first order in small gradients. So, special attention is paid to the local collision integral. The explicit expressions (11), (12) for the local collision integral are obtained. In the well-known case of a non-dissipative system with no external field, our result coincides with the well-known Landau collision integral [2].

On the basis of the local second-order collision integral it is shown that if the correlation functions of the external stochastic field and the function γ (see (14)) are even functions of the coordinate differences, the momentum of a spatially uniform Maxwellian gas is conserved.

Of course, the investigation of the energy time equation is also of great interest. This investigation will answer the question under which conditions the Maxwellian distribution function is an equilibrium one, and it will be made in another paper on the basis of the obtained second-order collision integral.

Acknowledgments

The author is grateful to Professor A. I. Sokolovsky for a fruitful discussion.

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Received 15.05.2016