UDC 530.1

S. F. Lyagushyn[1*], A. I. Sokolovsky[1], S. A. Sokolovsky[2], V. V. Yarlik[1]

[1]*Oles Honchar Dnipropetrovsk National University, Dnipro, Ukraine*
[2]*Pridneprovska State Academy of Civil Engineering and Architecture Sciences, Dnipro, Ukraine*
**e-mail: lyagush.new@gmail.com*

TO KINETICS OF QUANTUM SYSTEMS IN A MEDIUM

Kinetics of a quantum system immersed in an equilibrium medium and completely described by its statistical operator is investigated. Interaction of the system and medium is considered to be small. The case of a separable interaction that very common in the most applications is investigated in details. A kinetic equation for free point particle which interacts with a medium of point particles is obtained in terms of the Wigner distribution function of the particle. The derived kinetic equation does not assume that the Wigner distribution depends on coordinates weakly and therefore describes strong spatially inhomogeneous (localized) states of the particle. The elaborated theory is applied also to a system of two-level emitters and photons interacting with an equilibrium phonon medium. Phonons take into account consistently the motion of emitters in a crystalloid solid. A kinetic equation for the statistical operator of the system of the emitters and photons is obtained. It can be applied for investigating the effect of emitter motion on the superradiance phenomenon. The developed theory provides a sequential approach to the reduction of the problem dimensionality by introducing an effective interaction that corresponds to the modern trends in dynamics of complicated systems.

Key words: statistical operator, completely described quantum system, equilibrium medium, the basic kinetic equation, two-level emitters, effective interaction.

Досліджується кінетика повно описаної своїм статистичним оператором квантової системи, поміщеної до рівноважного середовища. Взаємодія системи і середовища вважається слабкою. Докладно досліджено випадок сепарабельної взаємодії, яка зустрічається у переважній більшості застосувань. У термінах вігнерівської функції розподілу отримано кінетичне рівняння для вільної точкової частинки, яка взаємодіє з середовищем із точкових частинок. Виведене кінетичне рівняння не передбачає слабкої залежності функції розподілу від координат, і тому описує також сильно неоднорідні (локалізовані) стани частинки. Розроблена теорія застосована до системи дворівневих випромінювачів і фотонів, взаємодіючих із рівноважним фононним середовищем. Фонони послідовно враховують рух випромінювачів у кристалічному твердому тілі. Отримано кінетичне рівняння для статистичного оператора системи випромінювачів і фотонів, що дає основу для дослідження впливу руху атомів на явище надвипромінювання. Розвинена теорія дає послідовний підхід до зменшення розмірності задачі шляхом запровадження ефективної взаємодії, що відповідає тенденціям динаміки складних систем.

Ключові слова: статистичний оператор, повно описана квантова система, рівноважне середовище, основне кінетичне рівняння, дворівневі випромінювачі, ефективна взаємодія.

Исследуется кинетика полно описанной своим статистическим оператором квантовой системы, помещенной в равновесную среду. Взаимодействие системы и среды считается слабым. Подробно исследован случай сепарабельного взаимодействия, которое встречается в преобладающем большинстве применений. В терминах вигнеровской функции распределения получено кинетическое уравнение для свободной точечной частицы, которая взаимодействует со средой из точечных частиц. Выведенное кинетическое уравнение не предполагает слабой зависимости функции распределения от координат, и поэтому описывает сильнонеоднородные (локализованные) состояния частицы. Разработанная теория применена к системе двухуровневых излучателей и фотонов, взаимодействующих с равновесной фононной средой. Фононы последовательно учитывают движение излучателей в кристаллическом твердом теле. Получено кинетические уравнение для статистического оператора системы излучателей и фотонов. Его можно применить для исследования влияния движения излучателей на явление сверхизлучения. Развитая теория дает последовательный подход к уменьшению размерности задачи за счет введения эффективного взаимодействия, что отвечает современным тенденциям исследования динамики сложных систем.

Ключевые слова: статистический оператор, полно описанная квантовая система, равновесная среда, основное кинетическое уравнение, двухуровневые излучатели, эффективное взаимодействие.

© S. F. Lyagushyn, A. I. Sokolovsky, S. A. Sokolovsky, V. V. Yarlik, 2016

1. Introduction

System (s) kinetics in medium (m) is an actual problem of the modern theory of nonequilibrium processes. Kinetics of a fully described system interacting with surrounding (environment, medium, bath, reservoir) is an important special case. Here the full description means describing a system by dint of a statistical operator (SO) $\rho_s(t)$. In fact in such problem it comes to the quantum mechanics of the open (dissipative) system (see, for example, the review in [1]; in literature kinetic equations for SO $\rho_s(t)$ are called "master equation", or "basic kinetic equation". In such investigations the surrounding is supposed to be a large system. It means that system number of degrees of freedom is much less than one for the medium. This assumption allows neglecting the feedback effect on the system environment. Most often additionally it is considered that the medium is equilibrium and its interaction with the system is weak.

In fact, it comes to the reduced description of a nonequilibrium state of a composite system $s+m$. The state of $s+m$ is fully described by its SO $\rho(t)$; this SO is not defined by the SO $\rho_s(t)$. In the literature different approximations are applied, but the problem of the transition to long times remains unsolved. The point is that the reduced description becomes possible after some time τ_0 has elapsed and consideration of this generally accepted idea is not trivial (the paper [2] may be mentioned as an example of lightweight attitude to this problem; there a system of spins (emitters) and phonons placed in a medium of photons).

The problem is solved in the method of reduced description (MRD) by Bogolyubov which is based on his idea of functional hypothesis (FH). The term "functional hypothesis" is traditional. Actually, its proof is begun with the development of the calculation procedure in some perturbation theory for mathematical objects introduced in it, and this calculation is done at each application of MRD (see, e.g., MRD review in [3]).

A substantial contribution to the development of the theory of such systems was done by the papers [4, 5], where on the basis of Bogolyubov MRD a general kinetic equation for the SO of a system placed into an environment was obtained. In [4] on the example of an equilibrium boson medium it is also shown that while taking into account the macroscopic nature of the environment (with using the thermodynamic limit transition), the system does not change the quasi-equilibrium state of the environment. Note that kinetics of a fully described quantum system in a nonequilibrium medium is studied in [6].

It should be emphasized that investigations of properties of fully described quantum systems interacting with macroscopic systems are essential for experiments and devices with nanosystems. The matter is that all work with such systems is performed with using macroscopic devices (in the simplest case just contacts).

Our paper is organized in such a way. In the Section 2 the result of the paper [1] for the case of an equilibrium medium is obtained simplistically. In the Section 3 the general theory is concretized for a separable interaction. The Section 4 is devoted to deriving the kinetic equation for a free particle placed to medium. In the Section 5 kinetics of a system of emitters, photons, and phonons taking into account emitter motion and forming equilibrium environment, is considered.

2. Kinetic equation for a fully described system placed to an equilibrium medium

Relying on the papers [4, 5], we shall consider a system $s+m$ consisting of the system s and equilibrium medium m. Let H_s and H_m be the spaces of states of s and

m. Then $\mathrm{H}_{s+m} \equiv \mathrm{H}_s \otimes \mathrm{H}_m$ is the state space of $s+m$. The statistical operator (SO) of the composite system $\rho(t)$ satisfies the quantum Liouville equation

$$\frac{\partial\rho(t)}{\partial t} = -\frac{i}{\hbar}[\hat{H},\rho(t)], \qquad \hat{H} = \hat{H}_s + \hat{H}_m + \hat{H}_{sm}, \tag{1}$$

where $\hat{H}$, $\hat{H}_s$, and $\hat{H}_m$ are Hamilton operators of systems $s+m$, s, and m, wherein $\hat{H}_{sm}$ describes the interaction between s and m. SO $\rho_s(t)$ of the system s is defined by the formula

$$\rho_s(t) = \tilde{\mathrm{S}}\mathrm{p}_m \rho(t) \tag{2}$$

where $\tilde{\mathrm{S}}\mathrm{p}_m$ is the trace over the medium states in the space $\mathrm{H}_s \otimes \mathrm{H}_m$ (it transforms the operators of $\mathrm{H}_s \otimes \mathrm{H}_m$ into the operators of H_s space and it should be distinguished from the usual trace Sp_m in the space H_m). The structure and properties of the operators $\tilde{\mathrm{S}}\mathrm{p}_m$ and Sp_m, their analogues $\tilde{\mathrm{S}}\mathrm{p}_s$ and Sp_s, and also the trace Sp in the space H_{s+m} are described in details in the book [7]. They are widely used in literature (see, for example, [8]).

Let us suppose that the system s and medium m interact weakly and during the time τ_0 in the system m an equilibrium state described with the Gibbs distribution corresponding to the temperature T_m is achieved. We shall construct the reduced description of $s+m$ in terms of $\rho_s(t)$.

According to (1) and (2), the SO $\rho_s(t)$ satisfies an equation

$$\frac{\partial\rho_s(t)}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s,\rho_s(t)] - \frac{i}{\hbar}\tilde{\mathrm{S}}\mathrm{p}_m[\hat{H}_{sm},\rho(t)] \tag{3}$$

We put the Bogolyubov reduced description method (see, e. g. [3]) in the basis of our consideration. The method is based on his idea of the functional hypothesis (FH). To obtain a closed equation for $\rho_s(t)$, we put down FH in the form

$$\rho(t) \xrightarrow[t\gg\tau_0]{} \rho(\rho_s(t,\rho_0)) \qquad (\rho_0 \equiv \rho(t=0)), \tag{4}$$

where the designation $\rho_s(t,\rho_0)$ for the parameter $\rho_s(t)$ at $t \gg \tau_0$ is introduced

$$\rho_s(t) \xrightarrow[t\gg\tau_0]{} \rho_s(t,\rho_0). \tag{5}$$

The FH means that the SO of the system after some time passing depends on time through the parameters that give the reduced description of the system state (in our case such parameter is $\rho_s(t,\rho_0)$ – the SO of the system s). Note that the SO $\rho(\rho_s)$ does not depend on the initial state ρ_0.

Suppose that SO $\rho(\rho_s(t,\rho_0))$ meets Eq. (1) not only at $t \gg \tau_0$, but also for all $t \ge 0$. It continues the reduced description parameter $\rho_s(t,\rho_0)$ to times $t \le \tau_0$, then at $t \ge 0$ this parameter satisfies a closed equation

$$\frac{\partial \rho_s(t,\rho_0)}{\partial t} = L_s(\rho_s(t,\rho_0)), \tag{6}$$

where the right side of the equation is expressed via $\rho(\rho_s)$ by the formula

$$L_s(\rho_s) \equiv -\frac{i}{\hbar}[\hat{H}_s,\rho_s] - \frac{i}{\hbar}\tilde{\mathrm{S}}\mathrm{p}_b[\hat{H}_{sm},\rho(\rho_s)]. \tag{7}$$

According to our previous consideration SO $\rho(\rho_s)$ satisfies the equations

$$\frac{\partial\rho(\rho_s)}{\partial\rho_s} L_s(\rho_s) = \mathbf{L}\rho(\rho_s), \qquad \tilde{\mathrm{S}}\mathrm{p}_m \rho(\rho_s) = \rho_s. \tag{8}$$

These equations are known [3] to have more than one solution. Therefore, they should be supplemented by a certain condition that is called a limit one, according to Bogolyubov. It should be a statement about the evolution of the system $s+m$ in the positive direction of time. We proceed from the following relation [1]

$$e^{t\mathbf{L}_0}\rho(\rho_s) \xrightarrow[t\gg\tau_0]{} e^{t\mathbf{L}_0}\rho_s w_m \qquad (\mathbf{L}_0 \equiv \mathbf{L}_s + \mathbf{L}_m), \tag{9}$$

where w_m is an equilibrium SO of the medium m

$$w_m \equiv e^{\frac{\Omega_m - \hat{H}_m + \mu_m \hat{N}_m}{T_m}}, \quad \mathrm{Sp}_m\, w_m \equiv 1 \quad (\mathbf{L}_m w_m = 0). \tag{10}$$

The left side of (9) is a SO of $s+m$ at the time t when evolution without interaction of s and m occurs, if $\rho(\rho_s)$ is taken as the initial SO of $s+m$. Formula (9) actually expresses the principle of spatial correlation weakening because $s+m$ evolution without interaction s with m "spreads" the systems in space. As the result, $\rho(\rho_s)$ is transformed into the product of the system SO ρ_s and the equilibrium SO w_m of the environment. Therefore, the relation (9) can be called the boundary condition of complete correlation weakening.

On this basis in papers [4, 5] a kinetic equation for the SO of a system in the medium is obtained with accuracy up to second-order (in interaction $\hat{H}_{sm}$) contributions inclusively

$$\frac{\partial\rho_s}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s,\rho_s] - \frac{i}{\hbar}\tilde{\mathrm{S}}\mathrm{p}_m[\hat{H}_{sm},\rho_s w_m] - \frac{i}{\hbar}\tilde{\mathrm{S}}\mathrm{p}_m[\hat{H}_{sm},\mathbf{K}\rho_s w_m], \tag{11}$$

where designated

$$\mathbf{K}\hat{A} = \frac{i}{\hbar}\int_{-\infty}^{0} d\tau \left\{[\hat{A},\hat{H}_{sm}(\tau)] - w_m \tilde{\mathrm{S}}\mathrm{p}_m[\hat{A},\hat{H}_{sm}(\tau)]\right\},$$

$$\hat{A}(t) \equiv e^{\frac{i}{\hbar}tH_0}\hat{A}e^{-\frac{i}{\hbar}tH_0}, \qquad \hat{H}_0 \equiv \hat{H}_s + \hat{H}_b \tag{12}$$

($\hat{A}$ is an arbitrary operator in H_{s+m}). The kinetic equation (11) gives the full description of a quantum system placed into the equilibrium medium. In other words, it is a basic equation of quantum mechanics in medium.

3. Kinetic equation for a system in an equilibrium medium: separable interaction

Further study of the kinetic equation (11) requires concretizing the interaction operator $\hat{H}_{sm}$. We assume that it has the structure

$$\hat{H}_{sm} = \sum_i \hat{s}_i \hat{m}_i \tag{13}$$

where operators $\hat{s}_i$ and $\hat{m}_i$ act, correspondingly, in the state spaces H_s and H_m of the systems s and m. This interaction is very common, as evidenced by our following examples, and called separable one. For interactions (13) calculations are simplified by using identities

$$[\hat{A}_s, \hat{B}_m] = 0, \quad [\hat{A}_s\hat{A}_m, \hat{B}_s\hat{B}_m] = \hat{A}_s\hat{B}_s[\hat{A}_m, \hat{B}_m] + [\hat{A}_s, \hat{B}_s]\hat{B}_m\hat{A}_m,$$
$$\tilde{\mathrm{S}}\mathrm{p}_m \hat{A}_s\hat{A}_m = \hat{A}_s \mathrm{Sp}_m \hat{A}_m, \quad \mathrm{Sp}_m[\hat{A}_m, \hat{B}_m] = 0. \tag{14}$$

On this basis, we have for the second contribution to (11)

$$\tilde{\mathrm{S}}\mathrm{p}_m[\hat{H}_{sm}, \rho_s w_m] = [\sum_i \hat{s}_i \overline{\hat{m}_i}, \rho_s] \qquad (\overline{\hat{A}_m} \equiv \mathrm{Sp}_m\, w_m \hat{A}_m). \tag{15}$$

Eq. (11) includes also the operator $\mathbf{K}$ presented in (12). It is determined with interaction in the Dirac picture

$$\hat{H}_{sm}(\tau) = \sum_i \hat{s}_i(\tau)\hat{m}_i(\tau), \quad \hat{s}_i(\tau) \equiv e^{\frac{i}{\hbar}\tau\hat{H}_s}\hat{s}_i e^{-\frac{i}{\hbar}\tau\hat{H}_s}, \quad \hat{m}_i(\tau) \equiv e^{\frac{i}{\hbar}\tau\hat{H}_m}\hat{m}_i e^{-\frac{i}{\hbar}\tau\hat{H}_m}. \tag{16}$$

and contains a contribution that is analogous to (15)

$$\tilde{\mathrm{S}}\mathrm{p}_m[\rho_s w_m, \hat{H}_{sm}(\tau)] = \sum_i [\rho_s, \hat{s}_i(\tau)]\, \mathrm{Sp}_m \hat{m}_i(\tau) w_m = [\rho_s, \sum_i \hat{s}_i(\tau)\overline{\hat{m}_i}] \tag{17}$$

(the trace $\mathrm{Sp}_m \hat{m}_i(\tau) w_m$ does not depend on τ)

Further calculations of the right side of the kinetic equation (11) give with taking into account two formulas from (14)

$$-\frac{i}{\hbar}\tilde{\mathrm{S}}\mathrm{p}_m[\hat{H}_{sm}, \mathbf{K}\rho_s w_m] = \frac{1}{\hbar^2}\sum_{i,i'}\int_{-\infty}^{0} d\tau \tilde{\mathrm{S}}\mathrm{p}_m \{[\hat{s}_i, \rho_s \hat{s}_{i'}(\tau)][w_m, \hat{m}_{i'}(\tau)]\hat{m}_i -$$
$$+[\hat{s}_i, [\rho_s, \hat{s}_{i'}(\tau)]]\hat{m}_{i'}(\tau) w_m \hat{m}_i - \overline{\hat{m}_{i'}}[\hat{s}_i, [\rho_s, \hat{s}_{i'}(\tau)]] w_m \hat{m}_i\}. \tag{18}$$

This expression can be written in the form

$$-\frac{i}{\hbar}\widetilde{\mathrm{S}}\mathrm{p}_b[\hat{H}_{sb},\hat{K}\rho_s w_b]=\frac{1}{\hbar^2}\sum_i\Big([\hat{s}_i,\rho_s\hat{A}_i]-[\hat{s}_i,\hat{B}_i\rho_s]\Big) \tag{19}$$

where designations

$$\hat{A}_i=\sum_{i'}\int_{-\infty}^{0}d\tau\hat{s}_{i'}(\tau)\langle\hat{m}_{i'}(\tau)\hat{m}_i\rangle\,,\qquad \hat{B}_i=\sum_{i'}\int_{-\infty}^{0}d\tau\hat{s}_{i'}(\tau)\langle\hat{m}_i\hat{m}_{i'}(\tau)\rangle \tag{20}$$

and correlation functions of medium

$$\langle\hat{A}_m\hat{B}_m\rangle=\overline{\hat{A}_m(\hat{B}_m-\overline{\hat{B}_m})} \tag{21}$$

are introduced.

With taking into account formulas (15) and (19), the kinetic equation (11) takes the form

$$\frac{\partial\rho_s}{\partial t}=-\frac{i}{\hbar}[\hat{H}_s^{\mathrm{eff}},\rho_s]+\frac{1}{\hbar^2}\sum_i\Big([\hat{s}_i,\rho_s\hat{A}_i]-[\hat{s}_i,\hat{B}_i\rho_s]\Big),\qquad \hat{H}_s^{\mathrm{eff}}\equiv\hat{H}_s+\sum_i\hat{s}_i\overline{\hat{m}_i} \tag{22}$$

where the right side is calculated with accuracy to second-order in $\hat{H}_{ms}$ contributions inclusively.

Note that the linearity of this equation is related to neglecting the feedback of the system on the environment (taking into account this impact was discussed in [4, 6]). Equation (22) contains the effective Hamilton operator $\hat{H}_s^{\mathrm{eff}}$ that accounts effects of the self-consistent field.

4. Kinetic equation for a point particle in an equilibrium medium

Let us consider the case of a point particle interacting with point particles of the environment through a pair potential

$$\hat{H}_s=\frac{\hat{\mathbf{p}}^2}{2m}+U(\hat{\mathbf{x}})\ ,\quad \hat{H}_{sm}=\sum_a\Phi(|\,\hat{\mathbf{x}}-\hat{\mathbf{x}}_a\,|)=\int d^3x'\Phi(|\,\hat{\mathbf{x}}-\mathbf{x}'\,|)\hat{n}(\mathbf{x}') \tag{23}$$

where the operator of particle density is introduced

$$\hat{n}(\mathbf{x})=\sum_a\delta(\mathbf{x}-\hat{\mathbf{x}}_a)\,. \tag{24}$$

It is convenient to put down the Hamilton operator of interaction $\hat{H}_{sm}$ in the form

$$\hat{H}_{sm}=\frac{1}{V}\sum_k\nu_k\,e^{i\mathbf{k}\hat{\mathbf{x}}}\hat{n}_{\mathbf{k}}\ ,\qquad \hat{n}_{\mathbf{k}}\equiv\int_V d^3x\,\hat{n}(\mathbf{x})\,e^{-i\mathbf{k}\mathbf{x}}\,, \tag{25}$$

where Fourier transformation was done

$$\nu_k\equiv\int_V d^3x\Phi(|\,\mathbf{x}\,|)\,e^{-i\mathbf{k}\mathbf{x}}\,,\qquad \Phi(|\,\mathbf{x}\,|)\equiv\frac{1}{V}\sum_k\nu_k\,e^{i\mathbf{k}\mathbf{x}} \tag{26}$$

(ν_k depends only on $\mathbf{k}$ module). Here the second formula implements the periodic continuation of the interaction potential that at a rather great volume of the system V accurately reproduces the potential $\Phi(|\mathbf{x}|)$. Wave vectors $\mathbf{k}$ take the values

$$\mathbf{k}_l = 2\pi\mathbf{n}_l / V^{1/3}, \qquad \mathbf{n}_l \in \mathbb{Z}$$

(it corresponds to using periodic boundary conditions) and sums over wave vectors are converted into integrals at great volumes

$$\frac{1}{V}\sum_{\mathbf{k}}(...) \xrightarrow[V\to\infty]{} \frac{1}{(2\pi)^3}\int d^3k(...)$$

The Hamilton operator (25) introduced above takes the form (17) at replacement

$$i \to \vec{k}\,, \quad \sum_i \to \sum_{\vec{k}}\,, \quad \hat{s}_i \to \frac{\nu_k}{V} e^{i\mathbf{k}\mathbf{x}}, \quad \hat{m}_i \to \hat{n}_{\mathbf{k}}\,. \tag{27}$$

To simplify the resulting kinetic equation (22), we note that averages of particle number density Fourier component products with the equilibrium statistical operator w_m have a property

$$\overline{\hat{n}^+_{\mathbf{k}_1}...\hat{n}^+_{\mathbf{k}_s}\hat{n}_{\mathbf{k}'_1}...\hat{n}_{\mathbf{k}'_{s'}}} \sim \delta_{\mathbf{k}_1+...+\mathbf{k}_s,\mathbf{k}'_1+...+\mathbf{k}'_{s'}} \quad (\hat{n}^+_{\mathbf{k}} = \hat{n}_{-\mathbf{k}}). \tag{28}$$

Hence, in particular, we have

$$\overline{\hat{m}_i} \to \overline{\hat{n}_{\mathbf{k}}} = \overline{n_0}\delta_{\mathbf{k},0}, \qquad \langle \hat{m}_{i'}(\tau)\hat{m}_i\rangle \to \left(\overline{\hat{n}^+_{\mathbf{k}}(\tau)\hat{n}_{\mathbf{k}}} - \overline{\hat{n}_0}^2\delta_{\mathbf{k},0}\right)\delta_{\mathbf{k}'=-\mathbf{k}},$$

$$[\hat{s}_i,\rho_s\hat{s}_{i'}(\tau)]\overline{\hat{m}_i}\,\overline{\hat{m}_{i'}} \to \frac{\nu_k^2}{V^2}[e^{i\mathbf{k}\mathbf{x}},\rho_s e^{i\mathbf{k}'\hat{\mathbf{x}}(\tau)}]\overline{n_0}\,\delta_{\mathbf{k},0}\,\overline{n_0}\,\delta_{\mathbf{k}',0} = 0$$

and therefore, in accordance with (23) we obtain

$$\hat{A}_i \to \frac{\nu_k}{V}\int_{-\infty}^{0} d\tau e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)}\langle \hat{n}^+_{\mathbf{k}}(\tau)\hat{n}_{\mathbf{k}}\rangle\,, \qquad \hat{B}_i \to \frac{\nu_k}{V}\int_{-\infty}^{0} d\tau e^{-i\vec{k}\,\hat{\vec{x}}(\tau)}\langle \hat{n}_k\hat{n}^+_k(\tau)\rangle\,.$$

In view of these formulas the kinetic equation (22) takes the form

$$\frac{\partial\rho_s}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s,\rho_s] + I(\rho_s)\,, \tag{29}$$

where the collision integral

$$I(\rho_s) \equiv \frac{1}{\hbar^2 V}\sum_k \nu_k^2\left([e^{i\mathbf{k}\hat{\mathbf{x}}},\rho_s\hat{A}_k] - [e^{i\mathbf{k}\hat{\mathbf{x}}},\hat{B}_k\rho_s]\right) \tag{30}$$

is introduced and designations

$$\hat{A}_{\mathbf{k}} = \frac{1}{V}\int_{-\infty}^{0} d\tau e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)}\langle \hat{n}^+_{\mathbf{k}}(\tau)\hat{n}_{\mathbf{k}}\rangle\,, \qquad \hat{B}_{\mathbf{k}} = \frac{1}{V}\int_{-\infty}^{0} d\tau e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)}\langle \hat{n}_{\mathbf{k}}\hat{n}^+_{\mathbf{k}}(\tau)\rangle\,. \tag{31}$$

are used. The obtained kinetic equation (29) is valid for a point particle in an arbitrary external field. In the case of free particles in the environment, in accordance with (16), the formulas (31) include

$$\hat{\mathbf{x}}_n(\tau) = e^{\frac{i}{\hbar}\tau\hat{H}_s}\hat{\mathbf{x}}_n e^{-\frac{i}{\hbar}\tau\hat{H}_s} = \hat{\mathbf{x}}_n + \frac{\tau}{m}\hat{\mathbf{p}}_n \,. \tag{32}$$

Proceed to further analysis of the kinetic equation (29). Instead of the statistical operator of a particle ρ_s, it is expedient to use the Wigner distribution function [4, 5] defined by the formula

$$\mathrm{f}_{\mathbf{p}}(\mathbf{x}) = \int d^3\mathbf{x}' \langle \mathbf{x} - \mathbf{x}'/2 \,|\, \rho_s \,|\, \mathbf{x} + \mathbf{x}'/2 \rangle \, e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}'} \,, \tag{33}$$

where $|\mathbf{x}\rangle$ is eigenvector of the coordinate operator $\hat{\mathbf{x}}_n$. The Wigner function is the closest to the classical one-particle distribution function, since it is real and quantities

$$w_1(\mathbf{x}) \equiv \langle \mathbf{x} \,|\, \rho_s \,|\, \mathbf{x} \rangle = \frac{1}{(2\pi\hbar)^3}\int d^3p\, \mathrm{f}_{\mathbf{p}}(\mathbf{x}) \,, \quad w_2(\mathbf{p}) \equiv \langle \mathbf{p} \,|\, \rho_s \,|\, \mathbf{p} \rangle = \frac{1}{(2\pi\hbar)^3}\int d^3x\, \mathrm{f}_{\mathbf{p}}(\mathbf{x}) \,, \tag{34}$$

are probability densities of particle coordinate and momentum values, correspondingly, ($|\mathbf{p}\rangle$ is an eigenvector of the particle momentum operator $\hat{\mathbf{p}}_n$). For the transition from momentum to coordinate representation and vice versa, the conditions of completeness and expression for $\langle \mathbf{x} \,|\, \mathbf{p} \rangle$ are useful

$$\int d^3x \,|\, \mathbf{x} \rangle\langle \mathbf{x} \,| = \hat{1} \,, \quad \int d^3p \,|\, \mathbf{p} \rangle\langle \mathbf{p} \,| = \hat{1} \,, \quad \langle \mathbf{x} \,|\, \mathbf{p} \rangle = \frac{1}{(2\pi\hbar)^{3/2}} e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}} \,.$$

Simple calculations show that the kinetic equation (29) can be written in terms of the Wigner distribution function (33) as

$$\frac{\partial \mathrm{f}_{\mathbf{p}}(\mathbf{x})}{\partial t} = -\frac{\mathbf{p}_l}{m}\frac{\partial \mathrm{f}_{\mathbf{p}}(\mathbf{x})}{\partial \mathbf{x}_l} + I_{\mathbf{p}}(\mathbf{x}, \mathrm{f}) \,. \tag{35}$$

Herewith the collision integral is given by the formulas

$$I_{\mathbf{p}}(\mathbf{x}, \mathrm{f}) \equiv \int d^3x' \langle \mathbf{x} - \mathbf{x}'/2 \,|\, I(\rho_s) \,|\, \mathbf{x} + \mathbf{x}'/2 \rangle \, e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}'} \,,$$

$$I(\rho_s) = \frac{1}{\hbar^2 V}\sum_k \int_{-\infty}^{0} d\tau g_{\mathbf{k}}(\tau)\left[e^{i\mathbf{k}\hat{\mathbf{x}}}\rho_s e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)} - e^{i\mathbf{k}\hat{\mathbf{x}}} e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)}\rho_s \right],$$

where designated

$$g_{\mathbf{k}}(\tau) \equiv \nu_k^2 [\langle \hat{n}_{\mathbf{k}}^{+}(\tau)\hat{n}_{\mathbf{k}} \rangle + \langle \hat{n}_{\mathbf{k}}\hat{n}_{\mathbf{k}}^{+}(\tau) \rangle] \,. \tag{36}$$

To calculate the required matrix elements, we note that a formula is valid

$$e^{-ik\left(\hat{\mathbf{x}} + \frac{t}{m}\hat{\mathbf{p}}\right)} = e^{i\frac{k^2\hbar}{2m}t} e^{-i\mathbf{k}\hat{\mathbf{x}}} e^{-i\frac{t}{m}\mathbf{k}\hat{\mathbf{p}}} \,, \tag{37}$$

which is a consequent of the Glauber identity

$$e^{\hat{A}+\hat{B}} = e^{\hat{A}} e^{\hat{B}} e^{-\alpha/2} \qquad (\alpha \equiv [\hat{A},\hat{B}]).$$

On the basis of the relation (37), we consistently find the matrix elements

$$\langle \mathbf{x} | e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)} | \mathbf{p} \rangle = e^{\frac{k^2\hbar}{2m}\tau} e^{-i\mathbf{k}\mathbf{x}} e^{-i\frac{\tau}{m}\mathbf{k}\mathbf{p}} \langle \mathbf{x} | \mathbf{p} \rangle ,$$

$$\langle \mathbf{x} | e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)} | \mathbf{x}' \rangle = e^{i\frac{k^2\hbar}{2m}\tau} e^{-i\mathbf{k}\mathbf{x}} \delta\left(\mathbf{x} - \mathbf{x}' - \mathbf{k}\frac{\hbar\tau}{m} \right),$$

which allow to calculate the contributions to the collision integral

$$\int d^3x' \langle \mathbf{x} - \mathbf{x}'/2 | e^{i\mathbf{k}\hat{\mathbf{x}}} \rho_s e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)} | \mathbf{x} + \mathbf{x}'/2 \rangle \, e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}'} = \mathrm{f}_{\mathbf{p}-\mathbf{k}\hbar}(\mathbf{x} + \mathbf{k}\frac{\hbar}{2m}\tau) e^{\frac{i}{\hbar}[\varepsilon_{\mathbf{p}-\mathbf{k}\hbar} - \varepsilon_{\mathbf{p}}]} ,$$

$$\int d^3x' \langle \mathbf{x} - \mathbf{x}'/2 | e^{i\mathbf{k}\hat{\mathbf{x}}} e^{-i\mathbf{k}\hat{\mathbf{x}}(\tau)} \rho_s | \mathbf{x} + \mathbf{x}'/2 \rangle \, e^{\frac{i}{\hbar}\mathbf{p}\mathbf{x}'} = \mathrm{f}_{\mathbf{p}}(\mathbf{x} - \mathbf{k}\frac{\hbar}{2m}\tau) e^{\frac{i}{\hbar}[\varepsilon_{\mathbf{p}-\mathbf{k}\hbar} - \varepsilon_{\mathbf{p}}]} .$$

As a result, with taking into account these formulas the collision integral acquires the final view

$$I_{\mathbf{p}}(\mathbf{x},\mathrm{f}) = \frac{1}{\hbar^2 V} \sum_k \int_{-\infty}^{0} d\tau g_{\mathbf{k}}(\tau) \left[\mathrm{f}_{\mathbf{p}-\mathbf{k}\hbar}(\mathbf{x} + \mathbf{k}\frac{\hbar\tau}{2m}) - \mathrm{f}_{\mathbf{p}}(\mathbf{x} - \mathbf{k}\frac{\hbar\tau}{2m}) \right] e^{\frac{i}{\hbar}(\varepsilon_{\mathbf{p}-\mathbf{k}\hbar} - \varepsilon_{\mathbf{p}})\tau} . \tag{38}$$

The kinetic equation (35) with collision integral (38) completely describes the dynamics of a free quantum particle placed in equilibrium environment. Therefore, it can be considered as the basic equation of quantum mechanics in such a situation. It is non-local in space and in this sense may describe strong nonuniform states of the particle. First time such equation for an electron (polaron) in the phonon medium were obtained in [4].

5. Kinetic equation for emitter system

Let us consider emitters in a crystal; their oscillating motion will be described in terms of phonons (system m). The internal degrees of freedom of emitters, photons, and the interaction between them in the absence of oscillations will be assumed as a system s. The Hamilton operator of $s+m$ system can be written, according to [9], in the form

$$\hat{H} = \hat{H}_m + \hat{H}_s + \hat{H}_{ms} , \qquad \hat{H}_s \equiv \sum_{n,s} \left(E_0 \hat{\mathbf{R}}_{nsz} - 2\hat{\mathbf{R}}_{nsx} \mathbf{d}_{sl} \hat{\mathbf{E}}_l(\mathbf{x}^o_{ns}) \right) + \sum_{\alpha,\mathbf{k}} \hbar\omega_k c^+_{\alpha\mathbf{k}} c_{\alpha\mathbf{k}} ,$$

$$\hat{H}_m = \hat{H}_{m0} + \hat{H}_{m,\mathrm{int}} , \qquad \hat{H}_{ms} \equiv -2\sum_{n,s} \hat{\mathbf{R}}_{nsx} \mathbf{d}_{sl} \left(\hat{\mathbf{E}}_l(\hat{\mathbf{x}}_{ns}) - \hat{\mathbf{E}}_l(\mathbf{x}^o_{ns}) \right),$$

$$\hat{\mathbf{x}}_{ns} = \mathbf{x}^o_{ns} + \hat{\mathbf{u}}_{ns} \qquad (\hat{\mathbf{u}}_{ns} \sim \mu \ll 1). \tag{39}$$

Here $\hat{\mathbf{x}}_{ns}$ is a radius-vector operator of the s-th particle in the n-th cell of a lattice, $\mathbf{x}^o_{ns}$ is an equilibrium position of this particle, μ is a parameter determining the smallness of particle displacements $\hat{\mathbf{u}}_{ns}$. The summand $\hat{H}_{m0}$ в $\hat{H}_m$ means the Hamilton operator of free photons; the contribution $\hat{H}_{\mathrm{int}}$ describes phonon interaction and can be expanded in a series in μ powers

$$\hat{H}_{m0} = \sum_{\lambda,\mathbf{q}} \hbar\omega_{\lambda\mathbf{q}} a^{+}_{\lambda\mathbf{q}} a_{\lambda\mathbf{q}} \sim \mu^0 , \qquad \hat{H}_{m,\mathrm{int}} = \hat{H}^{(1)}_m + \hat{H}^{(2)}_m + O(\mu^3) . \tag{40}$$

$\hat{\mathbf{E}}(\mathbf{x})$ in (39) stands for the operator of transversal electric field

$$\hat{\mathbf{E}}(\mathbf{x}) = \sum_{\alpha\mathbf{k}} \left(\frac{2\pi\omega_k \hbar}{V} \right)^{1/2} \{ \mathbf{e}_{\alpha}(\mathbf{k}) c_{\alpha\mathbf{k}} e^{i\mathbf{k}\cdot\mathbf{x}} + h.c. \} , \tag{41}$$

$\hat{\mathbf{R}}_{nsl}$ are Dicke operators, $\mathbf{d}_{sl}$ is a dipole moment of the noted particle in the lattice cell.

Displacement operators in standard notations are expressed by the formula

$$\hat{\mathbf{u}}_{nsl} = \sum_{\lambda\mathbf{q}} \frac{\hbar^{1/2}}{(2m_s \omega_{\lambda\mathbf{q}} N)^{1/2}} \{ \mathbf{e}_{\lambda sl}(\mathbf{q}) a_{\lambda\mathbf{q}} e^{i\mathbf{k}\mathbf{a}_n} + h.c. \} \tag{42}$$

(see, e.g., [10]). Note that our idea of taking into account emitter oscillations in the lattice is close to the approach proposed in [11] for the theory of ferromagnets. In the theory of the emitter system the same approach was discussed in [12].

We assume phonon subsystem to be equilibrium with the Gibbs SO

$$w_m = e^{\frac{\Omega_m - \hat{H}_m}{T_m}} , \qquad \mathrm{Sp}_m \, w_m = 1 .$$

With taking into account displacement smallness, the interaction operator $\hat{H}_{ms}$ has the structure of interaction (13), which is expected in the general theory developed by us

$$\hat{H}_{ms} = \sum_{ns} \hat{s}_{nsl} \hat{\mathbf{u}}_{nsl} + \sum_{ns} \hat{s}_{nsll'} \hat{\mathbf{u}}_{nsl} \hat{\mathbf{u}}_{nsl'} + O(\mu^3) , \tag{43}$$

where denoted

$$\hat{s}_{nsl_1} = -2\hat{\mathbf{R}}_{nsx} \mathbf{d}_{sl} \frac{\partial \hat{\mathbf{E}}_l(\mathbf{x}^o_{ns})}{\partial \mathbf{x}^o_{nsl_1}} , \qquad \hat{s}_{nsl_1l_2} = -\hat{\mathbf{R}}_{nsx} \mathbf{d}_{sl} \frac{\partial^2 \hat{\mathbf{E}}_l(\mathbf{x}^o_{ns})}{\partial \mathbf{x}^o_{nsl_1} \partial \mathbf{x}^o_{nsl_2}} . \tag{44}$$

Given the accuracy of the kinetic equation (22) we should put in it

$$\begin{gathered}
\sum_i \hat{s}_i \overline{\hat{m}_i} \to \sum_{ns} (\hat{s}_{nsl} \overline{\hat{\mathbf{u}}_{nsl}}^{(0)} + \hat{s}_{nsl} \overline{\hat{\mathbf{u}}_{nsl}}^{(1)}) + \sum_{ns,n's'} \hat{s}_{nsll'} \overline{\hat{\mathbf{u}}_{nsl} \hat{\mathbf{u}}_{nsl'}}^{(0)} , \\
\hat{A}_i \to \sum_{n's'} \int_{-\infty}^{0} d\tau \, \hat{s}_{n's'l'}(\tau) \overline{\hat{\mathbf{u}}_{n's'l'}(\tau) [\hat{\mathbf{u}}_{nsl} - \overline{\hat{\mathbf{u}}_{nsl}}^{(0)}]}^{(0)} , \\
\hat{B}_i \to \sum_{n's'} \int_{-\infty}^{0} d\tau \, \hat{s}_{n's'l'}(\tau) \overline{[\hat{\mathbf{u}}_{nsl} - \overline{\hat{\mathbf{u}}_{nsl}}^{(0)}] \hat{\mathbf{u}}_{n's'l'}(\tau)}^{(0)} ,
\end{gathered} \tag{45}$$

where $\overline{A_m}^{(n)} = \mathrm{Sp}_m \, w^{(n)}_m \hat{A}_m$ means the contribution of the n-th order in μ to the average $\overline{\hat{A}_m} = \mathrm{Sp}_m \, w_m \hat{A}_m$. Averages are calculated by dint of the thermodynamic theory of perturbations and Wick-Bloch-de Dominicis theorem [3]. We should take into account phonon interaction structure defined by the contribution of the first order describing three-photon processes. The contributions to the operator $\hat{H}^{(1)}_m$ have the form

$$\hat{H}_m^{(1)} = \frac{1}{N^{1/2}} \sum_{123} \{\Phi_1(12,3) a_1^+ a_2^+ a_3 + h.c.\} \sum_p \delta_{\mathbf{q}_1+\mathbf{q}_2-\mathbf{q}_3,\mathbf{b}_p} + \ldots \tag{46}$$

where conventional reduced notations of the type $a_i = a_{\alpha_i \mathbf{q}_i}$ are used (see, e.g., [3]). On this basis we have

$$\overline{\hat{\mathbf{u}}_{snl}}^{(0)} = 0, \qquad \overline{\hat{\mathbf{u}}_{snl}}^{(1)} \equiv \mathbf{u}_{sl}, \qquad \overline{\hat{\mathbf{u}}_{nsl}\hat{\mathbf{u}}_{nsl'}}^{(0)} = w_{sll'},$$
$$\overline{\hat{\mathbf{u}}_{snl}(\tau)\hat{\mathbf{u}}_{s'n'l'}}^{(0)} = g_{sl,s'l'}(n-n',\tau) \qquad (\overset{*}{g}_{sl,s'l'}(n-n',\tau) = g_{s'l',sl}(n'-n,-\tau)) \tag{47}$$

where functions $\mathbf{u}_{sl}$, $w_{sll'}$, $g_{sl,s'l'}(n-n',\tau)$ are introduced; they are determined by the thermodynamic theory of perturbations and not presented here. As a result, the kinetic equation (25) for the SO $\rho(t)$ of a system containing emitters and photons in equilibrium phonon environment takes the form

$$\frac{\partial \rho}{\partial t} = -\frac{i}{\hbar}[\hat{H}_s^{\text{eff}}, \rho] + \frac{1}{\hbar^2} \sum_{nsl} [\hat{s}_{nsl}, \rho \hat{A}_{nsl} - \hat{A}_{nsl}^+ \rho] \tag{48}$$

where denoted

$$\hat{H}_s^{\text{eff}} = \hat{H}_s + \sum_{ns} (\hat{s}_{nsl}\mathbf{u}_{sl} + \hat{s}_{nsll'} w_{sll'}), \qquad \hat{A}_{nsl} = \sum_{n's'} \int_{-\infty}^{0} d\tau\, \hat{s}_{n's'l'}(\tau) g_{s'l',sl}(n'-n,\tau). \tag{49}$$

Formulas (44) and (49) show that equilibrium phonon medium modifies the interaction between emitters and photons, implementing effective electric field that acts on the s-th particle in the lattice cell

$$\hat{\mathbf{E}}_s(\mathbf{x}) = \hat{\mathbf{E}}(\mathbf{x}) + \frac{\partial \hat{\mathbf{E}}(\mathbf{x})}{\partial \mathbf{x}_l} \mathbf{u}_{sl} + \frac{1}{2} \frac{\partial^2 \hat{\mathbf{E}}(\mathbf{x})}{\partial \mathbf{x}_l \partial \mathbf{x}_{l'}} w_{sll'}. \tag{50}$$

Formulas (45) indicate that the effective interaction (49) is a self-consistent field effect. The second term in the kinetic equation (48) gives some renormalization of the emitter-photon interaction too but also a dissipative contribution to the dynamics of the system.

6. Conclusions

In our paper a kinetic equation for a fully described quantum system placed into equilibrium environment is derived. Its right side is found in the approximation of the weak interaction between the system and medium and is calculated with accuracy to second-order contributions, inclusively. The case of a separable interaction between the system and medium is studied in detail; such interaction is implemented in most cases. For example, the spin-phonon interaction in solids has such a structure since particle oscillations in the lattice are small (we mean the nonlinear in displacements effects).

The paper is based on the Bogolyubov method of reduced description. It allows parsing the question of transition in the equations for the parameters of the reduced description to long times in the only way; in the literature it is done with additional assumptions (see examples in [1, 2, 8, 12]). In addition, the reduced description method leads to Markov kinetic equations. This is natural in problems of kinetics of a fully described system in the environment that is independent of time, since non-locality of kinetic equations in time emerges due to narrowing the list of system parameters.

The developed general theory is applied to the kinetics of a free point particle in a dense medium consisting of identical point particles. The potential interaction between the particle and the environment is considered to be small. The derived kinetic equation is written in terms of the Wigner distribution function of the particle. It accurately accounts for the effects of non-locality associated with the influence of spatial nonuniformity of the system on collisions.

The theory is applied for studying the impact of the motion of emitters (two-level atoms) on their interaction with photons. The situation in a crystalline solid is investigated under the assumption of weak interaction between emitters and phonons that describe their motion. Interaction weakness is associated with smallness of emitter oscillations. The consistent theory that takes into account effects of many-phonon processes is built. In particular, in the examined approximation three-phonon processes are considered. It is proved that emitter motion leads to renormalization of the interaction between emitters and photons.

References

1. **Weiss, U.** Quantum Dissipative Systems [Text] / U. Weiss. – Singapore: World Scientific, 2008. – 507 p.
2. **Bogolubov, N. N. (Jr.).** Variational master equation approach to dynamics of magnetic moments [Text] / N. N. Bogolubov (Jr.), A. V. Soldatov. – Dubna: JINR, 2016. – Preprint E17-2016-8. – 11 p.
3. **Akhiezer, A. I.** Methods of Statistical Physics [Text] / A. I. Akhiezer, S. V. Peletminskii. – Oxford: Pergamon, 1981. – 367 p.
4. **Sokolovsky, S. A.** Kineticheskoye uravneniye dlya silno neodnorodnykh sostoyaniy elektrona v kristalle [Text] / S. A. Sokolovsky, I. M. Chernenko // Visnik Dnipropetrovs'kogo Unìversitetu. Serìa Fìzika, Radìoelektronika. – 2003. – Issue 10. – P. 152 – 159.
5. **Sokolovsky, S. A.** Toward polaron kinetics in the Bogoliubov reduced description method [Text] / S. A. Sokolovsky // Theoretical and Mathematical Physics. – 2011. – Vol. 168, No. 2. – P. 1150 – 1164.
6. **Sokolovsky, S. A.** Osnovnoye kineticheskoye uravneniye dlya sistemy v neravnovesnoy srede [Text]/ S. A. Sokolovsky // Visnik Dnipropetrovs'kogo Unìversitetu. Serìa Fìzika, Radìoelektronika. – 2004. – Issue 12. – P. 78 – 84.
7. **Sokolovs'ky, O. Y.** Dodatkovi rozdily kvantovoyi teoriyi. Navchal'nyy posibnyk [Text] / O. Y. Sokolovsky, S. F. Lyagushyn. – Dnipropetrovs'k: RVV DNU, 2006. – 116 p.
8. **Bogolubov, N. N.** Aspects of Polaron Theory [Text}/ N. N. Bogolubov, N. N. Bogolubov (Jr.). – Singapore: World Scientific, 2008. – 180 p.
9. **Lyagushyn, S.F.** To the problem of the Hamiltonian form for a system of two-level atoms interacting with electromagnetic field [Text] / S.F. Lyagushyn, A. I. Sokolovsky // Visnik Dnipropetrovs'kogo Unìversitetu. Serìa Fìzika, Radìoelektronika. – 2015. – V. 23. – P. 54 – 59.
10. **Davydov, A. S.** Teoriya tviordogo tela [Text] / A. S. Davydov. – M.: Nauka, 1976. – 639 p.
11. **Akhiezer, A. I.** Spin waves / A. I. Akhiezer, V. G. Bar'yakhtar, S. V. Peletminskii. – Amsterdam: North-Holland, 1968. – 369 p.
12. **Bogolyubov, N. N. (ml.)** Sverkhizlucheniye [Text] / N. N. Bogolyubov (ml.), A. S. Shumovskiy. – Dubna: OIYaI, 1987. – 88 p.

Received 22.05.2016