UDC 537.862

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PHASED ANTENNA ARRAY ANALYSIS WITH SCHWARZ ALTERNATING METHOD

In the paper applying Schwarz alternating method to solving the problem of electromagnetic wave diffraction on a linear phased antenna array scanning in *H*-plane is considered. The initial problem is reduced to solving Fredholm integral equation of the second kind by the simple iteration method. Expressions for reflection coefficient in *i*-order iteration are obtained. An application range of Schwarz alternating method for this problem is obtained. An optimal iteration method, which extends the application range of Schwarz method, is proposed. Investigation of optimal iteration method convergence at different number of modes is performed. Reflection factor value at different wall thickness is obtained. The comparison of obtained results with known ones is carried out. A diffraction problem for a phased antenna array with dielectric-filled waveguides is solved. Relations between the reflection factor and scan angle at fixed permittivity values of dielectric filling are obtained. The applicability for such kind of problems is confirmed.

Keywords: integral equations, Green functions, Schwarz alternating method, phased antenna array, electromagnetic wave diffraction**.**

У роботі розглянуто застосування методу Шварця для розв'язання задачі дифракції електромагнітної хвилі на лінійній фазованій антенній решітці, скануючій у *H*-площині. Дифракційну задачу зведено до розв'язання інтегрального рівняння Фредгольма другого роду методом послідовних наближень. Отримано вирази для коефіцієнта відбиття падаючої хвилі в наближенні *i*-порядку. Встановлено межі застосування розглянутого варіанта методу Шварця. Запропоновано метод оптимальної ітерації, що розширює межі застосування методу Шварця. Проведено дослідження збіжності методу оптимальної ітерації за врахування різної кількості типів хвиль. Отримано значення коефіцієнта відбиття падаючої хвилі при різних значеннях товщини стінок хвилеводів решітки, а також проведено порівняння з уже відомими результатами. Розв'язано задачу про антенну решітку з хвилеводами, повністю заповненими діелектриком. Отримано залежності коефіцієнта відбиття падаючої хвилі від кута сканування за різних значень діелектричної проникності середовища, що заповнює хвилеводи. Підтверджено застосовність алгоритму до задач такого типу.

Ключові слова: інтегральні рівняння, функції Гріна, метод Шварця, фазована антенна решітка, дифракція електромагнітної хвилі.

В работе рассмотрено применение метода Шварца для решения задачи дифракции электромагнитной волны на линейной фазированной антенной решетке, сканирующей в *H*-плоскости. Дифракционная задача сведена к решению интегрального уравнения Фредгольма второго рода методом последовательных приближений. Получены выражения для коэффициента отражения падающей волны в приближении *i*-порядка. Установлены, границы применимости рассматриваемого варианта метода Шварца. Предложен метод оптимальной итерации, расширяющий границы применимости метода Шварца. Проведено исследование сходимости метода оптимальной итерации при различном числе учитываемых типов волн. Получено значение коэффициента отражения падающей волны при различных значениях толщины стенок волноводов решетки, а также проведено сравнение с уже известными результатами. Решена задача об антенной решетке с волноводами, полностью заполненными диэлектриками. Получены зависимости коэффициента отражения падающей волны от угла сканирования при различных значениях диэлектрической проницаемости среды, заполняющей волноводы. Подтверждена применымость алгоритма к задачам такого типа.

Ключевые слова: интегральные уравнения, функции Грина, метод Шварца, фазированная антенная решетка, дифракция электромагнитной волны.

1. Introduction

The analysis of characteristics of phased antenna arrays (PAA) with all features of electromagnetic processes is a difficult task. The integral equation method is one of the effective approaches to solving these problems. In paper [1] the application of the integral equation method for the analysis of waveguide phased antenna arrays is considered; numerical results of the calculations and the characteristics of antenna arrays of different types are presented. Also, one of the effective methods is the integral equation method for partially overlapping regions that provides the entire field domain to be sliced on two simple overlapping regions, for which the solution of the problem is known [2]. A similar approach is used in the Schwarz alternating method. This method is used for solving differential equations which satisfy the maximum principle. For example, such problems include the calculation of critical wave lengths of regular waveguide structures with non-coordinate cross-sections [3]. However, it is of interest to investigate the application range of the Schwarz alternating method in solving an inhomogeneous Helmholtz equation. For example, in paper [4] a problem of electromagnetic wave scattering on a concentric waveguide junction is solved by Schwarz alternating method with using tensor Green functions. Thus, the investigation of application range of the Schwarz method to solving waveguide problems is worth great attention.

2. Formulation of the problem

The Schwarz alternating method is an iterative method to find a solution of partial differential equations on a domain which is the union of two overlapping subdomains, by solving the equation on each of the subdomains in turn.

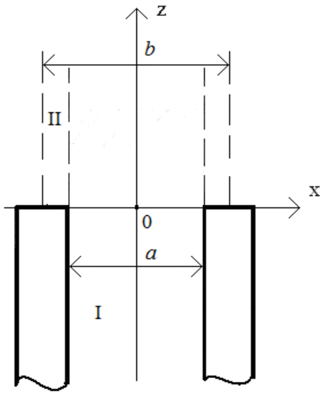


Fig.1. Unit cell of infinite parallel plate phased antenna array.

We consider an infinite parallel plate linear phased antenna array scanning in the *H*-plane with a finite waveguide wall thickness. Let us assume that the cells of the PAA are excited with equal amplitudes and phases that vary linearly. The fields in all periodic cells are the same and the phase changes by a constant value from cell to cell. Therefore, the field will be determined in a unit cell that is located at the origin. We will use the E_y field component that satisfies the two-dimensional Helmholtz equation as unknown function in integral equations, the boundary conditions for the electric field tangential components on perfectly conducting metal walls and the radiation condition [1]:

$$\Delta E_y(x,z) + k^2 E_y(x,z) = -J(x,z). \tag{1}$$

The solution of this problem by Schwarz method consists in converting the differential equation to the system of integral equations for two overlapping regions and solving it by the simple iteration method [5].

At first, we divide the whole field definition domain of the selected PAA cell into two partially overlapping regions (Fig. 1). Region I: $-a/2 \le x \le a/2$, $-\infty \le z \le \infty$. Region II: $-b/2 \le x \le b/2$, $0 \le z \le \infty$. The planar waveguide H_{10} wave is excited in region I at $z \to -\infty$. Suppose that the Green functions of regions I and II are known. Then we can make a system of integral representations of fields for each region with Green's second identity:

$$E_y^{\mathrm{I}}(x,z) = E_{ex}(x,z) + \int_0^{\infty} E_y^{\mathrm{II}}\left(-\frac{a}{2}, z'\right)\frac{\partial}{\partial x'} G^{\mathrm{I}}\left(x, z; -\frac{a}{2}, z'\right) dz' - \\ - \int_0^{\infty} E_y^{\mathrm{II}}\left(\frac{a}{2}, z'\right)\frac{\partial}{\partial x'} G^{\mathrm{I}}\left(x, z; \frac{a}{2}, z'\right) dz', \tag{2}$$

$$E_y^{\mathrm{II}}(x,z) = \int_0^{\infty} E_y^{\mathrm{I}}(x'',0)\frac{\partial}{\partial z''} G^{\mathrm{II}}(x', z'; x'', 0)\, dx''. \tag{3}$$

The Green function of region I is represented as a series of normalized orthogonal waveguide eigenfunctions:

$$G^{\mathrm{I}}(x,z;x',z') = \sum_{q=1}^{\infty} \varphi_q(x)\varphi_q(x') \frac{1}{2j\gamma_q} \exp\left(-j\gamma_q |z - z'|\right). \tag{4}$$

Here: $\varphi_q(x) = \sqrt{\frac{2}{a}} \sin\frac{q\pi}{a}\left(x + \frac{a}{2}\right)$ is a normalized orthogonal waveguide eigenfunction, $\gamma_q = -j\sqrt{\left(\frac{q\pi}{a}\right)^2 - k^2}$, $k=2\pi$. Because of the periodic character of the PAA excitement the Green function of the second region is represented as a series of “Floquet” harmonics [1]:

$$G^{\mathrm{II}}(x,z;x',z') = \sum_{q=1}^{\infty} \psi_q(x)\psi_q^*(x') \frac{1}{j\Gamma_m} \exp(-j\Gamma_m z)\,\mathrm{sh}(j\Gamma_m z'). \tag{5}$$

Here: $\psi_q(x) = \sqrt{\frac{1}{b}} \exp\left(j\frac{U_m}{b}x\right)$, symbol “*” denotes complex conjugation, $U_m = kb\sin\theta + 2m\pi$, $\Gamma_m = -j\sqrt{\left(\frac{kb\sin\theta + 2m\pi}{b}\right)^2 - k^2}$. Incident wave is a waveguide wave with unit amplitude composed with excluding diffraction on discontinuity:

$$E_{ex}(x,z) = \varphi_1(x)\exp(-j\gamma_1 z). \tag{6}$$

Assuming the values of each unknown function at the intersection of regions are equal each other and fixing coordinates of source and observation points, we substitute the equation (3) into (2) and obtain a Fredholm integral equation of the second kind in respect to the field function of the first region:

$$E_y^{\mathrm{I}}(x,z) = E_{ex}(x,z) + \int_{-\frac{a}{2}}^{\frac{a}{2}} E_y^{\mathrm{I}}(x'',0) K(x,z;x'',0)\, dx'', \tag{7}$$

$$K(x,z;x'',z'')=\int_0^{\infty}\left[\frac{\partial}{\partial z''}G^{\mathrm{II}}\left(-\frac{a}{2},z';x'',0\right)\frac{\partial}{\partial x'}G^{\mathrm{I}}\left(x,z;-\frac{a}{2},z'\right)-\right.$$
$$\left.-\frac{\partial}{\partial z''}G^{\mathrm{II}}\left(\frac{a}{2},z';x'',0\right)\frac{\partial}{\partial x'}G^{\mathrm{I}}\left(x,z;\frac{a}{2},z'\right)\right]dz' \quad (8)$$

Performing the integration in the integral equation kernel (8) allows us to obtain the following expression for it:

$$K(x,z;x'',z'')=\sum_{m=-\infty}^{\infty}\sum_{q=1}^{\infty}\varphi_q(x)\exp(j\gamma_q z)\psi_m^*(x'')C_{qm}\,, \quad (9)$$

$$C_{qm}=\sqrt{\frac{2}{ab}}\frac{-q\pi}{2a\gamma_q(\gamma_q+\Gamma_m)}\left[\exp\left(-j\frac{aU_m}{2b}\right)-\exp\left(j\frac{aU_m}{2b}\right)\cos(q\pi)\right]. \quad (10)$$

Then, we solve the integral equation (7) directly using a simple iteration method. Exciting wave function (6) is used as a zero-order approximation.

In order to obtain an expression for the reflection factor of H_{10} incident wave, it is needed to represent the unknown function as a sum of incident and reflected waves:

$$E_y^{\mathrm{I}\langle 1\rangle}(x,z)=\varphi_1(x)\exp(-j\gamma_1 z)+\sum_{q=1}^{\infty}R_q\varphi_q(x)\exp(j\gamma_q z). \quad (11)$$

Using (11) in the left side of the field definition expression and equating the coefficients at equal φ_q allow us to obtain the value of the reflection factor in each approximation order. Thus, the reflection factor at i-order approximation is represented by the following expression, where infinite numbers of modes are reduced to their finite values:

$$R_{10}^{\langle i\rangle}=\sum_{n=1}^{i}V_1^{\langle n\rangle}\,. \quad (12)$$

Here: $D_{qm}=\int_{-\frac{a}{2}}^{\frac{a}{2}}\varphi_q(x'')\psi_m^*(x'')dx''$, $V_p^{\langle n\rangle}=\sum_{m=-M}^{M}\sum_{q=1}^{Q}V_q^{\langle n-1\rangle}D_{qm}C_{pm}$, $R_{10}^{\langle 0\rangle}=0$, $V_p^{\langle 0\rangle}=1$, i=1,2,3…∞; M, and Q are maximum numbers of considered modes in "Floquet" and waveguide regions correspondingly.

3. Numerical results

According to the given algorithm, the program for computing numerical values of the incident wave reflection factor modulus and phase was compiled. On the base of the obtained results an investigation of the method convergence and its application range was performed as well as a comparison of the results with the known values obtained by the method of partially overlapping regions (MPOR) [2].

The results of the method convergence on each approximation step and comparison with known data are shown in table 1. The antenna array parameters are: b/λ=0.5714; $sin\theta$=0.05; $(b-a)/b$=0; 0.02; 0.063. Numbers of considered higher order modes are M=16, Q=32 at each approximation step.

The numerical experiment show that the convergence of series (12) decreases significantly when wall thickness takes such values at which the reflection factor modulus reaches values $|R_{10}|\geq 0.45$ and the kernel rate $\|K\|>1.12$. Further increasing of the wall thickness value makes the simple iteration method inapplicable to solve this problem.

Table 1

The results of the method convergence at each approximation step

i	$(b-a)/b$=0		$(b-a)/b$=0.02		$(b-a)/b$=0.063	
	||K||=1.0164		||K||=1.1		||K||=1.3859	
	Modulus	Phase	Modulus	Phase	Modulus	Phase
1	0.44525	138.99	0.51085	138.19	0.75049	138.25
2	0.3574	165.15	0.39451	168.33	0.52793	-174.28
3	0.32867	151.43	0.35131	149.5	0.39834	136.38
4	0.35842	155.24	0.40182	155.51	0.6079	160.77
5	0.34223	155.36	0.37016	155.86	0.39392	163.84
6	0.34755	154.60	0.38173	154.23	0.51254	150.61
7	0.34695	155.09	0.38058	155.39	0.49421	163.56
8	0.34636	154.90	0.37859	154.89	0.44892	154.95
9	0.34687	154.94	0.38039	154.99	0.5062	157.58
10	0.34664	154.95	0.37949	155.04	0.46431	159.46
11	0.3467	154.94	0.37974	154.98	0.48094	156.04
12	0.3467	154.94	0.37977	155.01	0.48453	158.84
13	0.34669	154.94	0.37969	155	0.47139	157.47
14	0.3467	154.94	0.37974	155	0.48413	157.53
15	0.3467	154.94	0.37972	155	0.47674	158.2
	Exact solution		MPOR			
	0.347	155.9	0.37836	155.11	0.47169	157.99

In order to obtain a convergent solution, the simple iteration method (SIM) can be transformed into the optimal iteration method (OIM). With this aim in view the left side in the equation (7) was moved to the right and the resulting expression was multiplied by a factor β. Then the unknown function $E_y(x,z)$ was added to both sides of the resulting equation. Thus, an expression for $E_y(x,z)$ in i-order approximation takes the form

$$E_y^{1\langle i\rangle}(x,z)=\beta E_{ex}(x,z)+(1-\beta)E_y^{1\langle i-1\rangle}(x,z)+\int_{-\frac{a}{2}}^{\frac{a}{2}} E_y^{1\langle i-1\rangle}(x'',0)K(x,z;x'',0)dx'' . \qquad (13)$$

After all the necessary changes, the expressions for the reflection factor in i-order approximation takes the form

$$R_{10}^{\langle i\rangle}=R_{10}^{\langle i-1\rangle}+\sum_{n=1}^{i} A_{in}(1-\beta)^{(i-n)}V_1^{\langle n\rangle} . \qquad (14)$$

Here $V_p^{\langle n\rangle}=\beta\sum_{m=-M}^{M}\sum_{q=1}^{Q}V_q^{\langle n-1\rangle}D_{qm}C_{pm}$, $R_{10}^{\langle 0\rangle}=0$, $V_p^{\langle 0\rangle}=1$, $A_{in}=A_{(i-1)(n-1)}+A_{(i-1)n}$, $A_{11}=1$, i=1, 2, 3…∞.

The iterative process can be convergent to an approximate result with a minimum relative error by choosing a coefficient β to be equal to the reciprocal value of kernel rate.

Let us consider the effect of the number of considered modes on the solution convergence and accuracy for simple and optimal iteration methods. PAA parameters are: b/λ=0.5714, $\sin\theta$=0.05, $t=(b-a)/b$=0. The convergence for both methods at different numbers of considered modes M and Q at each approximation step is given in table 2.

Table 2 shows that calculations with a few numbers of considered modes cause minimal computational accuracy and require maximum number of iterations in order to obtain a convergent solution. At larger values of M the iteration process is convergent at $I \le 15$.

Table 2

The convergence of SIM and OIM at each approximation step at b/λ=0.5714, $sin\theta$=0.05, $t=(b-a)/b$=0, M=2;12;26, $Q=2M$

i	$\lvert R_{10}\rvert$, OIM			$\lvert R_{10}\rvert$, SIM		
	M=2	M=12	M=26	M=2	M=12	M=26
1	0.49199	0.43585	0.41934	0.44525	0.45365	0.45395
2	0.34459	0.37836	0.37951	0.35588	0.37693	0.37746
3	0.30517	0.3208	0.32717	0.31159	0.31432	0.31446
4	0.37146	0.35345	0.34936	0.34995	0.35967	0.35988
5	0.32001	0.34817	0.34895	0.33518	0.34644	0.34677
6	0.34127	0.34489	0.34592	0.33599	0.34421	0.34447
7	0.33969	0.34734	0.34713	0.33846	0.34825	0.34853
8	0.33402	0.3468	0.34715	0.33685	0.34644	0.34674
9	0.33929	0.34662	0.34697	0.33729	0.34660	0.34688
10	0.33661	0.34680	0.34704	0.33735	0.34689	0.34718
11	0.33719	0.34675	0.34704	0.33723	0.34669	0.34698
12	0.33756	0.34674	0.34703	0.33729	0.34674	0.34703
13	0.33706	0.34675	0.34703	0.33728	0.34676	0.34705
14	0.33737	0.34675	0.34704	0.33727	0.34674	0.34703
15	0.33727	0.34675	0.34703	0.33728	0.34675	0.34704
16	0.33725	0.34675	0.34703	0.33727	0.34675	0.34704
17	0.33730	0.34675	0.34703	0.33727	0.34675	0.34703
18	0.33726	0.34675	0.34703	0.33728	0.34675	0.34703
19	0.33728	0.34675	0.34703	0.33727	0.34675	0.34703
20	0.33728	0.34675	0.34703	0.33728	0.34675	0.34703

Table 3 shows calculation results for values of the reflection factor modulus, phase and required runtime at different values of M and Q at i=15. PAA parameters are: b/λ=0.5714, $sin\theta$=0.05, $t=(b-a)/b$=0. Table 3 shows, that increase of a number of considered modes leads to increasing an accuracy of the calculation, however, it also increasing a runtime required for obtaining each value of the reflection factor. Thus, values M=16, Q=32 were chosen for further calculations.

Table 4 shows numerical results obtained at different values of scan angle and waveguide wall thickness using SIM and OIM along with known results obtained using the method of partially overlapping regions. PAA parameters are: b/λ=0.5714, $t=(b-a)/b$=0.063; 0.12. The number of iterations is i = 15. The number of considered modes amounts M = 16, Q = 32.

Table 3

Reflection factor modulus, phase and required runtime at different values of M and Q at i=15, b/λ=0.5714, $sin\theta$=0.05, $t=(b-a)/b$=0

M	Q	OIM			SIM		
		Modulus	Phase	Runtime, s.	Modulus	Phase	Runtime, s.
2	4	0.33727	154.27	0.094	0.33728	154.26	0.031
4	8	0.34424	155.04	0.172	0.34423	155.04	0.156
6	12	0.34575	155.32	0.5	0.34575	155.32	0.516
8	16	0.34632	155.46	1.141	0.34632	155.46	1.125
10	20	0.34659	155.54	2.156	0.34659	155.54	2.172
12	24	0.34675	155.60	3.703	0.34675	155.60	3.688
14	28	0.34684	155.64	5.828	0.34684	155.64	5.843
16	32	0.34690	155.67	8.609	0.34690	155.67	8.719
18	36	0.34695	155.70	12.235	0.34695	155.70	12.671
20	40	0.34698	155.72	17	0.34698	155.72	16.781
22	44	0.34700	155.73	22.063	0.34700	155.73	22.906
24	48	0.34702	155.75	28.703	0.34702	155.74	29.422
26	52	0.34703	155.76	37.156	0.34704	155.76	36.516

Table 4

Reflection factor modulus and phase, obtained with different methods at b/λ=0.5714

t	$\sin\theta$	MPOR		SIM		OIM	
		$\lvert R_{10}\rvert$	Phase	$\lvert R_{10}\rvert$	Phase	$\lvert R_{10}\rvert$	Phase
0,063	0.05	0.47176	158.01	0.47845	158.27	0.47164	158
	0.20	0.46258	156.65	0.46543	156.57	0.46246	156.64
	0.40	0.42858	151.62	0.42839	151.59	0.42846	151.61
	0.60	0.34548	139.23	0.34534	139.21	0.34534	139.21
	0.70	0.24574	124.07	0.24556	124.06	0.24556	124.06
0,12	0.05	0.80654	172.94	>1	–	0.80645	172.93
	0.20	0.80173	172.61	>1	–	0.80163	172.61
	0.40	0.78317	171.47	>1	–	0.78304	171.47
	0.60	0.73269	169.1	>1	–	0.73254	169.1
	0.70	0.66365	167.66	>1	–	0.66349	167.66

The results shown in Table 4 confirm that OIM allows us to obtain a problem solution when SIM is inapplicable. Thus, the OIM-algorithm enlarges the application ranges of Schwarz alternating method.

4. The phased antenna array with dielectric-filled waveguides

Now we assume that the PAA cell has a waveguide region at $-a/2 \le x \le a/2$, $-\infty \le z \le 0$ filled with uniform isotropic dielectric matter with permittivity ε. It is needed to find the reflection factor of incident wave inside waveguide. The Schwarz algorithm described above can be used to solve this problem. The integral equation for the unknown function has the form similar to Eq. (7). “Floquet” region remains unchanged, thus, Green function for it is described by Eq. (5). The Green function for parallel plate waveguide is represented in a “sourcewise” form:

$$G^{\mathrm{I}}(x,z;x',z') = \sum_{q=1}^{\infty} \varphi_q(x)\varphi_q(x') f_q(x,x'). \tag{15}$$

The series of normalized orthogonal waveguide eigenfunctions φ_q is unchanged. It is necessary to find only function depending on the longitudinal coordinates of source (*z)* and observation (*z’)* points. To do this, we form the equation of electromagnetic wave propagation in an infinite parallel plate waveguide for each subregion with the presence of a dielectric filling. It should be noted that the source points are located at z’ > 0.

$$\begin{cases} f_q^{\mathrm{I}}(x,x') = t\exp\left(j\gamma_q^{\mathrm{I}} z\right) \quad at \quad -\infty \le z \le 0, \\ f_q^{\mathrm{II}}(x,x') = \dfrac{1}{2j\gamma_q^{\mathrm{II}}}\exp\left(-j\gamma_q^{\mathrm{II}}\lvert z-z'\rvert\right) + r\exp\left(-j\gamma_q^{\mathrm{II}} z\right) \quad at \quad 0 \le z \le \infty \end{cases} \tag{16}$$

A set of equations for the unknown coefficients t and r can be formed by equating functions from (16) to each other at the interface z=0. In this formulation of the problem observation point takes values $z \le 0$. Thus, it is needed to determine only coefficient t and to use only first equation from system (16) in order to build the Green function. An unknown function has the following form

$$f_q^{\mathrm{I}}(x,x') = \frac{1}{j\gamma_q^{\mathrm{I}} + j\gamma_q^{\mathrm{II}}}\exp\left(-j\gamma_q^{\mathrm{II}} z'\right)\exp\left(j\gamma_q^{\mathrm{I}} z\right). \tag{17}$$

Here $\gamma_q^{\mathrm{I(II)}} = -j\sqrt{\left(\dfrac{q\pi}{a}\right)^2 - \left(k^{\mathrm{I(II)}}\right)^2}$, $k^{\mathrm{II}}=2\pi$, $k^{\mathrm{I}} = 2\pi\sqrt{\varepsilon}$.

The kernel of the integral equation is described by Eq. (9). Coefficient C_{qm} is determined by the following expression

$$C_{qm} = \sqrt{\frac{2}{ab}} \frac{-q\pi}{a\left(\gamma_q^I + \gamma_q^{II}\right)\left(\gamma_q + \Gamma_m\right)} \left[\exp\left(-j\frac{aU_m}{2b}\right) - \exp\left(j\frac{aU_m}{2b}\right)\cos(q\pi) \right]. \qquad (18)$$

Exciting wave is a waveguide wave with unit magnitude composed with a considering of the wave reflection on medium interface

$$E_{ex}(x,z) = \varphi_1(x)\exp\left(-j\gamma_1^{\mathrm{I}} z\right) + R_{10}^{\langle 0 \rangle}\varphi_1(x)\exp\left(j\gamma_1^{\mathrm{I}} z\right). \qquad (19)$$

Here $R_{10}^{\langle 0 \rangle}$ — the reflection factor of the incident wave H_{10} in a parallel plate waveguide from the interface between two media with different permittivity, which is determined by the expression:

$$R_{10}^{\langle 0 \rangle} = \frac{\gamma_1^{\mathrm{I}} - \gamma_1^{\mathrm{II}}}{\gamma_1^{\mathrm{I}} + \gamma_1^{\mathrm{II}}}. \qquad (20)$$

In order to obtain the value of reflection factor the unknown function of electrical field in a waveguide is represented in the form of the expression (11). Solving the integral equation (7) we obtain an expression for the reflection factor at *i*-order approximation:

$$R_{10}^{\langle i \rangle} = \left(1 + R_{10}^{\langle 0 \rangle}\right)\sum_{n=1}^{i} V_1^{\langle n \rangle}. \qquad (21)$$

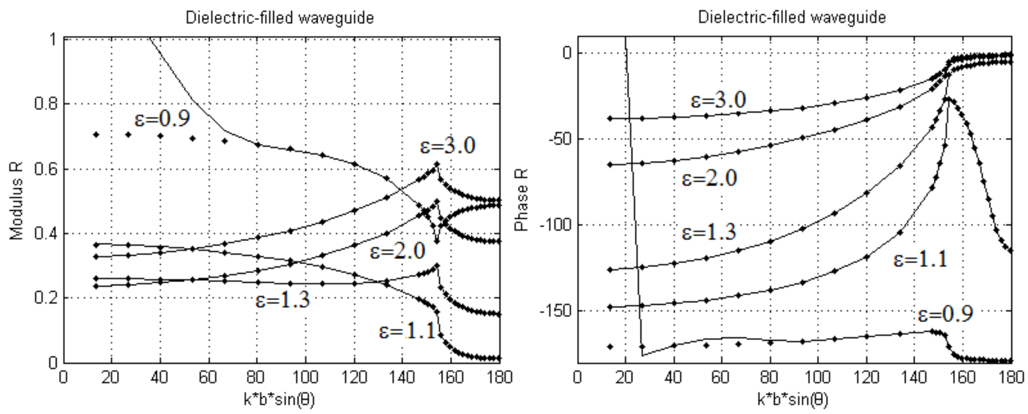
Here $V_p^{\langle n \rangle} = \sum_{m=-M}^{M} \sum_{q=1}^{Q} V_q^{\langle n-1 \rangle} D_{qm} C_{pm}$, $V_p^{\langle 0 \rangle} = 1$, i=1,2,3…∞.

The same expressions for reflection factor can be found for OIM through solving an Eq. (13):

$$R_{10}^{\langle i \rangle} = R_{10}^{\langle i-1 \rangle} + \left(1 + R_{10}^{\langle 0 \rangle}\right)\sum_{n=1}^{i} A_{in}(1-\beta)^{(i-n)} V_1^{\langle n \rangle}. \qquad (22)$$

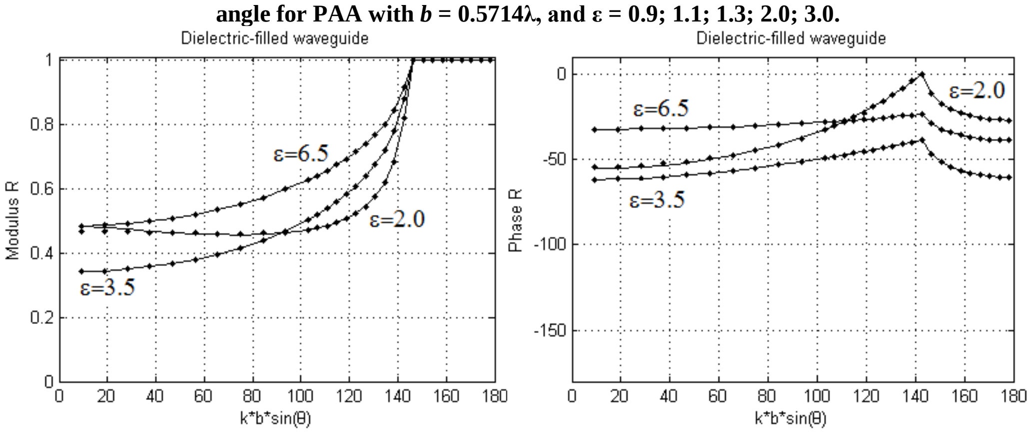
Here: $V_p^{\langle n \rangle} = \beta \sum_{m=-M}^{M} \sum_{q=1}^{Q} V_q^{\langle n-1 \rangle} D_{qm} C_{pm}$, $V_p^{\langle 0 \rangle} = 1$, $A_{in} = A_{(i-1)(n-1)} + A_{(i-1)n}$, $A_{11} = 1$, i=1,2,3…∞.

On the basis of obtained results the value of the reflection factor was calculated for PAA with parameters b = 0.5714λ and b = 0.4λ, $t = (b - a)/b$ = 0.063. The permittivity of the dielectric filling took values from 0.9 to 6.5. The values of the dielectric filling permittivity were chosen in order to allow only one of waveguide modes to be propagating inside the waveguide. The dependence of the reflection factor modulus and phase against the value of scan angle is plotted in Fig. 2 for PAA with b = 0.5714λ, ε = 0.9; 1.1; 1.3; 2.0; 3.0 and in Fig. 3 for PAA with b = 0.4λ, ε = 2.0; 3.5; 6.5. Calculations were made using SIM (solid line) and OIM (dots). It was assumed that M = 16, Q = 32, I = 15. The results obtained in paper [1] for similar PAA is shown in Fig. 4.



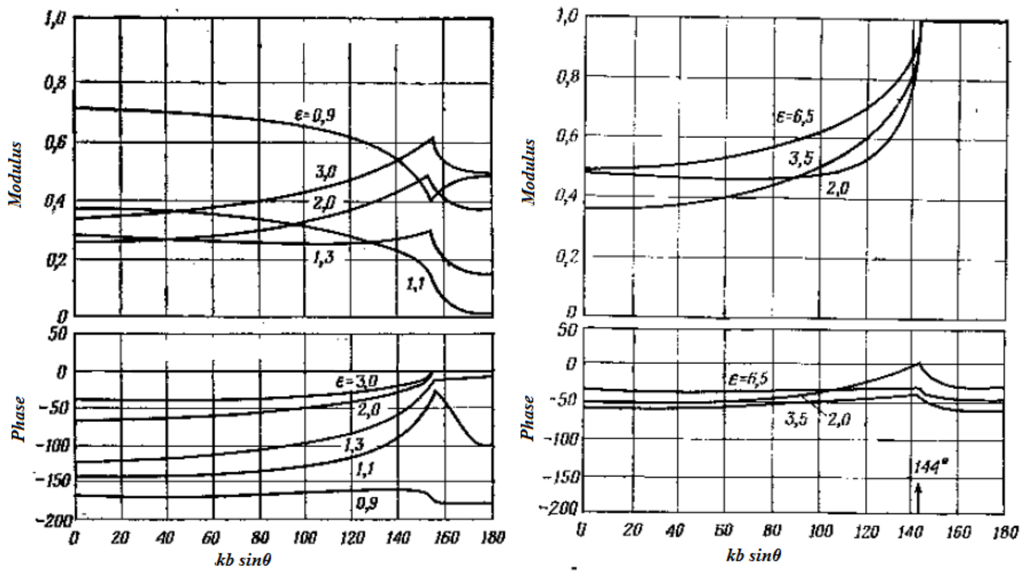
(a) **(b)**

Fig. 2. The dependence of the reflection factor modulus (a) and phase (b) against the value of scan angle for PAA with *b* = 0.5714λ, and ε = 0.9; 1.1; 1.3; 2.0; 3.0.



(a) **(b)**

Fig. 3. The dependence of the reflection factor modulus (a) and phase (b) against the value of scan angle for PAA with *b* = 0.4λ, and ε = 2.0; 3.5; 6.5.



(a) **(b)**

Fig. 4. Known results from paper [1]: *b* = 0.5714λ (a), *b* = 0.4λ (b).

5. Conclusions

Applying the Schwarz alternating method to electromagnetic diffraction problems is considered in this paper. Expressions for H_{10}−type incident wave reflection factor in unit cell of infinite PAA with finite waveguide wall thickness are obtained. Numerical experiments allow finding the dimensions of discontinuity, at which the simple iteration method becomes inapplicable and gives incorrect results. In this case, an optimal iteration method giving a convergent solution for this problem at larger discontinuity dimensions, was used. The investigation of convergence was performed for both methods at different numbers of considered modes in waveguide and Floquet regions. As a result, the convenient number of modes and maximal iteration order that give a solution with maximal accuracy are obtained. The results are compared with known ones for this problem that shows the correctness of the built algorithm. These results are used to obtain a solution for infinite PAA of parallel plate waveguides with dielectric filling. Obtained dependences of the reflection factor versus scan angle at different permittivity of waveguide filling are shown as plots. For this solution the number of considered waves in waveguide region is Q = 32, in “Floquet” region M = 16 and iteration order i = 15. It should be noted that at $\varepsilon = 0.9$ and scan angles $kb\sin\theta \leq 100^{\circ}$ a simple iteration method does not allow to obtain a convergent solution. This fact is well shown in Fig. 2. The comparison of the obtained results with known ones from paper [1] shows correctness of the built algorithm and its applicability for solving such kind of electromagnetic scattering problems.

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Received 25.05.2015.