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HYDRODYNAMIC MODES OF THE LANDAU KINETIC EQUATION IN THE ABSENCE OF RELAXATION

Hydrodynamics of completely ionized two-component electron-ion plasma is investigated on the basis of the Landau kinetic equation. The relaxation of component temperatures and velocities is considered to be finished. The investigation follows the idea of the Bogolyubov functional hypothesis. The consideration is based on a perturbation theory in small gradients of the reduced description parameters with additional account of the smallness of the electron-to-ion mass ratio. Hydrodynamic equations for the reduced description parameters are built taking into account dissipative processes. A linearized theory in the vicinity of the equilibrium state is developed. Linearized hydrodynamic equations are obtained. The equation solutions are analyzed. The dispersion laws for the hydrodynamic modes of the system are obtained up to the second order of the perturbation theory in small wave vector. The effect of the Landau collision integral on the hydrodynamic modes of the system is investigated. The results can serve as basis for the investigation of relaxation phenomena in the vicinity of hydrodynamic states.

Keywords: completely ionized election-ion plasma, Landau kinetic equation, linearized theory, hydrodynamic equations, hydrodynamic modes.

На основі кінетичного рівняння Ландау досліджується гідродинаміка повністю іонізованої двокомпонентної електрон-іонної плазми. Релаксація швидкостей та температур компонент вважається завершеною. В основу розгляду покладено ідею функціональної гіпотези Боголюбова. Розгляд базується на теорії збурень за малими градієнтами параметрів скороченого опису з додатковим урахуванням малості відношення мас електрона та іона. Рівняння гідродинаміки для параметрів скороченого опису побудовано з урахуванням дисипативних процесів. Розвинуто лінеаризовану теорію в околі рівноважного стану. Отримано лінеаризовані рівняння гідродинаміки, їх розв'язки аналізуються. Дисперсійні закони для гідродинамічних мод системи отримано з точністю до другого порядку малості за малим хвильовим вектором. Вивчено вплив інтегралу зіткнень Ландау на гідродинамічні моди системи. Результати роботи можуть бути основою для вивчення релаксаційних процесів поблизу гідродинамічних станів.

Ключові слова: повністю іонізована електрон-іонна плазма, кінетичне рівняння Ландау, лінеаризована теорія, рівняння гідродинаміки, гідродинамічні моди.

На основе кинетического уравнения Ландау исследуется гидродинамика полностью ионизированной двухкомпонентной электрон-ионной плазмы. Релаксация скоростей и температур компонент считается завершенной. В основу рассмотрения положена идея функциональной гипотезы Боголюбова. Рассмотрение базируется на теории возмущений по малым градиентам параметров сокращенного описания с дополнительным учетом малости отношения масс электрона и иона. Уравнения гидродинамики для параметров сокращенного описания построены с учетом диссипативных процессов. Развита линеаризованная теория в окрестности равновесного состояния. Получены линеаризованные уравнения гидродинамики, их решения анализируются. Законы дисперсии для гидродинамических мод системы получены с точностью до второго порядка малости по малому волновому вектору. Изучено влияние интеграла столкновений Ландау на гидродинамические моды системы. Результаты работы могут служить основой для изучения релаксационных процессов вблизи гидродинамических состояний.

Ключевые слова: полностью ионизированная электрон-ионная плазма, кинетическое уравнение Ландау, линеаризованная теория, уравнения гидродинамики, гидродинамические моды.

1. Introduction

In his famous paper [1] Landau derived a kinetic equation for a completely ionized gas with Coulomb interaction, which is widely used in the kinetic theory of plasma. On the basis of the equation, hydrodynamics of completely ionized two-component plasma is investigated. This paper is concerned with the non-homogenous case where the relaxation of the component temperatures and velocities is finished. The consideration is based on the idea of the Bogolyubov functional hypothesis. In our previous papers [2,3] we also considered the plasma hydrodynamics, but those papers are devoted to the derivation of the distribution function and the kinetic coefficients of the system.

The aim of the present paper is to build the hydrodynamic modes of the Landau kinetic equation in the absence of relaxation. Usually, the plasma hydrodynamic modes are investigated on the basis of the Vlasov kinetic equation, but the collision integral is omitted there. The investigation of the hydrodynamic modes of a two-component plasma with taking into account the collision integral cannot be found in the literature. The problem of investigating the plasma hydrodynamic modes on the basis of the Landau equation is a model one; nevertheless, it is rather important because it describes the effect of the collision integral on the hydrodynamic modes. The results of the paper are also important for obtaining the hydrodynamic modes in the presence of relaxation [4,5].

The paper is organized as follows. In Sec.2 the basic equations of the theory and our previous results for the distribution functions are given. In Sec.3 hydrodynamic equations in terms of fluxes are obtained. In Sec.4 linearized hydrodynamic equations are obtained, and on the basis of these results in Sec.5 the hydrodynamic modes of the system are obtained.

2. The distribution functions and the basic equations of the theory

The well-known Landau kinetic equation for completely ionized electron-ion plasma has the form

$$\frac{\partial f_{ap}(x,t)}{\partial t} = -\frac{p_{an}}{m_a}\frac{\partial f_{ap}(x,t)}{\partial x_n} + I_{ap}\left(f(x,t)\right) \tag{1}$$

where $f_{ap}(x,t)$ is the distribution function of the a-th component of the plasma (a, b, c,... $= e,i$), I_{ap} is the Landau collision integral [1]. The Landau equation is a model one, but it adequately describes the role of the Coulomb interaction in the system at long distances. Therefore, it is widely used in the plasma theory.

As is known [6], the reduced description parameters in the one-velocity and one-temperature hydrodynamics can be chosen as the particle number densities of the components $n_a(x,t)$, the temperature $T(x,t)$ and the velocity $\upsilon_n(x,t)$ of the system. By definition, these parameters are introduced as follows:

$$\int f_{ap} d^3 p = n_a, \quad \pi_n \equiv \sum_a \int f_{ap} p_n d^3 p = \upsilon_n \rho, \quad \varepsilon \equiv \sum_a \int f_{ap} \varepsilon_{ap} d^3 p = \frac{3}{2} nT + \frac{1}{2}\rho\upsilon^2 \tag{2}$$

where π_n and ε are the total momentum and energy densities, respectively, ρ is the total mass density of the system ($\rho \equiv m_e n_e + m_i n_i$) and n is the total particle density of the system ($n \equiv n_e + n_i$); $\varepsilon_{ap} = p^2 / 2m_a$.

The investigation is based on the idea of the Bogolyubov functional hypothesis [6], which can be written in the form

$$f_{ap}\left(x,t\right)\xrightarrow[t\gg\tau_0]{} f_{ap}\left(x,\xi(t)\right) \qquad (3)$$

where the reduced description parameters are denoted as ξ_μ : $\xi_0=T$, $\xi_n=\upsilon_n$, $\xi_a=n_a$ ($\mu=0,n,a$). In (3) τ_0 is a time which is much shorter than the subsystem velocity and temperature relaxation times.

The dependence of the reduced description parameters on the coordinates is supposed to be weak, so the corresponding small parameter g is introduced:

$$\frac{\partial^s\xi_\mu\left(x\right)}{\partial x_{n_1}...\partial x_{n_s}}\sim g^s \qquad (g\ll 1). \qquad (4)$$

Also we use the additional smallness of the electron-to-ion mass ratio by introducing the small parameter

$$\sigma=\left(m_e/m_i\right)^{1/2}. \qquad (5)$$

The functions $f_{ap}\left(x,\xi\right)$ are found from (1), (3) up to the first order in the gradients of the parameters ξ_μ :

$$f_{ap}\left(x,\xi\right)=f_{ap}^{(0)}+f_{ap}^{(1)}+O\left(g^2\right). \qquad (6)$$

As usual, the functions $f_{ap}^{(0)}$ are Maxwellian ones

$$f_{ap}^{(0)}=w_{a,p\text{-}m_a\upsilon}, \qquad w_{ap}\equiv\frac{n_a\beta^{3/2}}{\left(2\pi m_a\right)^{3/2}}\exp\left(-\beta\varepsilon_{ap}\right) \qquad (\beta\equiv 1/T). \qquad (7)$$

The functions $f_{ap}^{(1)}$ are found in [2,3]. They have the form:

$$f_{ap}^{(1)}=w_{a,p-m_a\upsilon}\Bigg[p_n\sum_b\left(g_{a0}^{N_b}+g_{a1}^{N_b}\left(\frac{5}{2}-\beta\varepsilon_{ap}\right)\right)\frac{\partial n_b}{\partial x_n}+ +p_n\left(g_{a0}^T+g_{a1}^T\left(\frac{5}{2}-\beta\varepsilon_{ap}\right)\right)\frac{\partial T}{\partial x_n}+h_{nl}\left(p\right)g_{a0}^{\upsilon}\frac{\partial\upsilon_n}{\partial x_l}\Bigg]_{p\to p-m_a\upsilon} \qquad (8)$$

where $h_{nl}\left(p\right)=p_np_l-p^2\delta_{nl}/3$ and the coefficients $g_{a0}^{N_b}$, $g_{a1}^{N_b}$, g_{a0}^T, g_{a1}^T, g_{a0}^{υ} are calculated in the σ perturbation theory. They are rather lengthy, that is why they are not given here; they are given in [2,3]. The results of this section are important for obtaining the hydrodynamic modes that is the aim of the present paper.

3. Hydrodynamic equations

The hydrodynamic equations are the equations of the form

$$\frac{\partial\xi_\mu\left(x,t\right)}{\partial t}=L_\mu\left(x,f\left(\xi(t)\right)\right). \qquad (9)$$

According to [7], we should know them in order to obtain the hydrodynamic modes of the system. From (1) and (2) it can be shown that

$$\frac{\partial \rho_a}{\partial t} = -\frac{\partial \pi_{an}}{\partial x_n}, \qquad \frac{\partial \pi_n}{\partial t} = -\frac{\partial t_{nl}}{\partial x_l}, \qquad \frac{\partial \varepsilon}{\partial t} = -\frac{\partial q_n}{\partial x_n} \tag{10}$$

where ρ_a is the a-th component mass density ($\rho_a = m_a n_a$), π_{an} is the a-th component momentum density, t_{nl} and q_n are the total momentum and energy fluxes, respectively. The definition of these quantities is

$$\pi_{an} = \int f_{ap} p_n d^3 p, \qquad q_l = \sum_a \int d^3 p \varepsilon_{ap} \frac{p_l}{m_a} f_{ap}, \qquad t_{nl} = \sum_a \int d^3 p p_n \frac{p_l}{m_a} f_{ap}. \tag{11}$$

From (2) and (11) it can be obtained that

$$\varepsilon = \varepsilon^o + \frac{1}{2}\rho \upsilon^2, \qquad t_{nl} = t^o_{nl} + \rho \upsilon_n \upsilon_l, \qquad q_n = q^o_n + \upsilon_l t^o_{nl} + \upsilon_n \left(\varepsilon^o + \rho \frac{\upsilon^2}{2} \right),$$

$$\pi_{en} = j_n + \rho_e \upsilon_n, \qquad \pi_{in} = -j_n + \rho_i \upsilon_n, \qquad j_n = \pi^o_{en} \tag{12}$$

where ε^o, t^o_{nl}, q^o_n are the energy density, momentum flux and energy flux in the accompanying reference frame, respectively, and j_n is the diffusive flux:

$$\varepsilon^o = \frac{3}{2} nT, \qquad t^o_{nl} = \sum_a \int d^3 p p_n \frac{p_l}{m_a} f_{a, p+m_a \upsilon}, \qquad q^o_l = \sum_a \int d^3 p \varepsilon_{ap} \frac{p_l}{m_a} f_{a, p+m_a \upsilon},$$

$$j_n = \int f_{e, p+m_e \upsilon} p_n d^3 p. \tag{13}$$

From (10) and (12) it can be obtained, that

$$\frac{\partial \rho_e}{\partial t} = -\frac{\partial j_n}{\partial x_n} - \rho_e \frac{\partial \upsilon_n}{\partial x_n} - \upsilon_n \frac{\partial \rho_e}{\partial x_n}, \qquad \frac{\partial \rho_i}{\partial t} = \frac{\partial j_n}{\partial x_n} - \rho_i \frac{\partial \upsilon_n}{\partial x_n} - \upsilon_n \frac{\partial \rho_i}{\partial x_n},$$

$$\frac{\partial \upsilon_n}{\partial t} = -\frac{1}{\rho} \frac{\partial t^o_{nl}}{\partial x_l} - \upsilon_l \frac{\partial \upsilon_n}{\partial x_l}, \qquad \frac{\partial \varepsilon^o}{\partial t} = -\frac{\partial q^o_n}{\partial x_n} - t^o_{nl} \frac{\partial \upsilon_l}{\partial x_n} - \upsilon_n \frac{\partial \varepsilon^o}{\partial x_n} - \varepsilon^o \frac{\partial \upsilon_n}{\partial x_n}. \tag{14}$$

Our next step is to obtain hydrodynamic equations from (14). Although above we used the parameters n_a, υ_n, T as the reduced description paremeters, it is more convenient to use the total mass density ρ, the dimensionless parameter c, the velocity υ_n and the temperature T as reduced description parameters [6]; the definition of the parameter c is

$$\rho_e \equiv \rho c. \tag{15}$$

Notice that there is no contradiction here because ρ and c can be expressed in terms of n_a and vice versa:

$$\rho = m_e n_e + m_i n_i, \qquad c = \frac{m_e n_e}{m_e n_e + m_i n_i}, \qquad n_e = \frac{c\rho}{m_e}, \qquad n_i = \frac{\rho(1-c)}{m_i}. \tag{16}$$

From (14) and (15) it is easy to obtain the hydrodynamic equations:

$$\frac{\partial \rho}{\partial t} = -\rho \frac{\partial \upsilon_n}{\partial x_n} - \upsilon_n \frac{\partial \rho}{\partial x_n}, \qquad \frac{\partial c}{\partial t} = -\frac{1}{\rho}\frac{\partial j_n}{\partial x_n} - \upsilon_n \frac{\partial c}{\partial x_n}, \qquad \frac{\partial \upsilon_n}{\partial t} = -\frac{1}{\rho}\frac{\partial t^o_{nl}}{\partial x_l} - \upsilon_l \frac{\partial \upsilon_n}{\partial x_l},$$

$$\frac{\partial T}{\partial t} = \left[\left(\frac{\partial \varepsilon^o}{\partial T}\right)_{\rho c}\right]^{-1} \left\{ -\upsilon_n \left(\frac{\partial \varepsilon^o}{\partial T}\right)_{\rho c} \frac{\partial T}{\partial x_n} + \left(\left(\frac{\partial \varepsilon^o}{\partial \rho}\right)_{\rho T} \rho - \varepsilon^o\right)\frac{\partial \upsilon_n}{\partial x_n} - \right.$$

$$\left. -\frac{\partial q^o_n}{\partial x_n} - t^o_{nl}\frac{\partial \upsilon_l}{\partial x_n} + \left(\frac{\partial \varepsilon^o}{\partial c}\right)_{\rho T} \frac{1}{\rho}\frac{\partial j_n}{\partial x_n} \right\}. \tag{17}$$

The hydrodynamic equations (17) can be simplified. As known [6],

$$q^o_n = \chi j_n - \kappa \frac{\partial T}{\partial x_n}, \quad t^o_{nl} = p\delta_{nl} - \eta\left(\frac{\partial \upsilon_n}{\partial x_l} + \frac{\partial \upsilon_l}{\partial x_n} - \frac{2}{3}\delta_{nl}\frac{\partial \upsilon_m}{\partial x_m}\right) - \zeta \delta_{nl}\frac{\partial \upsilon_m}{\partial x_m} \tag{18}$$

where κ is the thermal conductivity of the system, χ is an additional kinetic coefficient, η and ζ are shear and bulk viscosities, respectively, and p is the pressure. The substitution of (18) into (17) can simplify the time equations for T and υ_n, but, for simplicity, we will make it after the linearization of the theory.

4. Linearized hydrodynamic equations

According to [7], we should linearize the theory in order to investigate the hydrodynamic modes of the system. The linearized theory is built as follows. The deviations of the reduced description parameters from their equilibrium values are assumed to be small:

$$\rho = \rho_0 + \delta\rho, \qquad c = c_0 + \delta c, \qquad T = T_0 + \delta T, \qquad \upsilon_n = \delta\upsilon_n \tag{19}$$

where we use the reference frame $\upsilon_{0n} = 0$, here and in what follows the subscript 0 means that the quantity is taken at equilibrium. The hydrodynamic equations are written up to the first order in the small deviations (19). Substituting (18) into (17), we obtain

$$\frac{\partial \delta\rho}{\partial t} = -\rho_0 \frac{\partial \upsilon_n}{\partial x_n}, \qquad \frac{\partial \delta c}{\partial t} = -\frac{1}{\rho_0}\frac{\partial \delta j_n}{\partial x_n},$$

$$\frac{\partial \delta\upsilon_n}{\partial t} = -\frac{1}{\rho_0}\left[\left(\frac{\partial p}{\partial T}\right)^0_{c\rho}\frac{\partial \delta T}{\partial x_n} + \left(\frac{\partial p}{\partial c}\right)^0_{T\rho}\frac{\partial \delta c}{\partial x_n} + \left(\frac{\partial p}{\partial \rho}\right)^0_{cT}\frac{\partial \delta\rho}{\partial x_n} - \right.$$

$$\left. -\eta \frac{\partial^2 \delta\upsilon_n}{\partial x_l \partial x_l} - \left(\frac{\eta}{3} + \zeta\right)\frac{\partial^2 \delta\upsilon_l}{\partial x_n \partial x_l}\right],$$

$$\frac{\partial \delta T}{\partial t} = \left[\left(\frac{\partial \varepsilon^o}{\partial T}\right)^0_{\rho c}\right]^{-1} \left\{ \left[\left(\frac{\partial \varepsilon^o}{\partial \rho}\right)^0_{cT}\rho_0 - \varepsilon^o_0 - p_0\right]\frac{\partial \delta\upsilon_n}{\partial x_n} + \right.$$

$$\left. + \left(\left(\frac{\partial \varepsilon^o}{\partial c}\right)^0_{\rho T}\frac{1}{\rho_0} - \chi_0\right)\frac{\partial \delta j_n}{\partial x_n} + \kappa_0 \frac{\partial^2 \delta T}{\partial x_n \partial x_n}\right\}. \tag{20}$$

From (6)–(8), (13) and (18) it can be obtained that

$$\zeta = 0\,, \quad p = nT\,, \quad j_n = n_e m_e T\left\{\sum_b g_{e0}^{N_b}\frac{\partial n_b}{\partial x_n} + g_{e0}^{T}\frac{\partial T}{\partial x_n}\right\},$$

$$q_n^o = T^2\sum_a\sum_b\left\{\frac{5}{2}n_a\left(g_{a0}^{N_b} - g_{a1}^{N_b}\right)\right\}\frac{\partial n_b}{\partial x_n} + T^2\sum_a\left\{\frac{5}{2}n_a\left(g_{a0}^{T} - g_{a1}^{T}\right)\right\}\frac{\partial T}{\partial x_n}\,. \tag{21}$$

Using (18) and (21), we can express χ and κ in terms of $g_{a0}^{N_b}$, $g_{a1}^{N_b}$, g_{a0}^{T}, g_{a1}^{T}:

$$\chi = \frac{T}{n_e m_e g_{e0}^{N_e}}\sum_a\left\{\frac{5}{2}n_a\left(g_{a0}^{N_e} - g_{a1}^{N_e}\right)\right\}, \quad \kappa = \chi n_e m_e T g_{e0}^{T} - T^2\sum_a\left\{\frac{5}{2}n_a\left(g_{a0}^{T} - g_{a1}^{T}\right)\right\}. \tag{22}$$

Finally, from (16), (20)-(22) and the explicit expressions for $g_{a0}^{N_b}$, $g_{a1}^{N_b}$, g_{a0}^{T}, g_{a1}^{T}, g_{a0}^{υ} [2,3] we obtain the linearized hydrodynamic equations in the form:

$$\frac{\partial\delta\rho}{\partial t} = -\rho_0\,\mathrm{div}\,\delta\upsilon_n\,, \quad \frac{\partial\delta c}{\partial t} = \gamma_\rho\Delta\delta\rho + \gamma_c\Delta\delta c + \gamma_T\Delta\delta T\,,$$

$$\frac{\partial\delta T}{\partial t} = \alpha_\rho\Delta\delta\rho + \alpha_c\Delta\delta c + \alpha_T\Delta\delta T - \frac{2}{3}T\,\mathrm{div}\,\delta\upsilon_n\,,$$

$$\frac{\partial\delta\upsilon_n}{\partial t} = -\beta_T\frac{\partial\delta T}{\partial x_n} - \beta_c\frac{\partial\delta c}{\partial x_n} - \beta_\rho\frac{\partial\delta\rho}{\partial x_n} + \eta_\upsilon\Delta\delta\upsilon_n + \frac{\eta_\upsilon}{3}\frac{\partial^2\delta\upsilon_l}{\partial x_n\partial x_l} \tag{23}$$

where the coefficients ρ_0, γ_ρ, γ_c, γ_T, α_ρ, α_c, α_T, β_T, β_c, β_ρ, η_υ are calculated in the σ perturbation theory and taken at equilibrium. The electroneutrality condition $n_{e0} = zn_{i0}$ (z is the ion charge number) is also taken into account during these calculations. The expressions for the above-mentioned coefficients are

$$\rho_0 = -n_i m_e\sigma^{-2} + O\left(\sigma^0\right), \quad \alpha_\rho = \frac{z\sigma^2}{m_e}\alpha_N + O\left(\sigma^3\right), \quad \alpha_c = \alpha_N n_i\sigma^{-2} + O\left(\sigma^{-1}\right),$$

$$\alpha_N = \frac{2zT^2}{3(z+1)}\left(\frac{5}{2}g_{e1}^{N_e(0)} - g_{e0}^{N_e(0)}\right), \quad \alpha_T = \frac{2T^2 z}{3(z+1)}\left\{\frac{5}{2}g_{e1}^{T(0)} - g_{e0}^{T(0)}\right\} + O(\sigma),$$

$$\gamma_\rho = -\frac{Tz^2}{m_e}g_{e0}^{N_e(0)}\sigma^4 + O\left(\sigma^5\right), \quad \gamma_c = -n_i zTg_{e0}^{N_e(0)} + O(\sigma),$$

$$\gamma_T = -zTg_{e0}^{T(0)}\sigma^2 + O\left(\sigma^3\right), \quad \beta_T = \frac{z+1}{m_e}\sigma^2 + O\left(\sigma^4\right), \quad \beta_c = \frac{T}{m_e} + O\left(\sigma^2\right),$$

$$\beta_\rho = \frac{T(z+1)}{n_i m_e^{\,2}}\sigma^4 + O\left(\sigma^6\right), \quad \eta_\upsilon = -T^2 g_{i0}^{\upsilon(1)} + O\left(\sigma^2\right) \tag{24}$$

where $g_{e0}^{N_e(0)}$, $g_{e1}^{N_e(0)}$, $g_{e0}^{T(0)}$, $g_{e1}^{T(0)}$ are the contribution of the order σ^0 to $g_{e0}^{N_e}$, $g_{e1}^{N_e}$, g_{e0}^{T}, g_{e1}^{T}, respectively, and $g_{i0}^{\upsilon(1)}$ is the contribution of the order σ^1 to g_{i0}^{υ}.

In what follows the dispersion laws for the hydrodynamic modes of the system are investigated on the basis of the linearized hydrodynamic equations (23) with the coefficients (24). To obtain these dispersion laws, we should take the Fourier transform of equations (23) and analyze the solutions of the obtained equations for the Fourier components of the reduced description parameters.

5. Hydrodynamic modes of the system

Let us take the Fourier transform

$$\xi_{\mu k}\left(k,t\right)=\int e^{-ik_n x_n}\delta\xi_{\mu}\left(x,t\right)d^3x\,,\quad \delta\xi_{\mu}\left(x,t\right)=\left(2\pi\right)^{-3}\int e^{-ik_n x_n}\xi_{\mu k}\left(k,t\right)d^3k \qquad (25)$$

where $\xi_{\mu}=\{\rho,\upsilon_n,T,c\}$. After taking this transform and choosing the reference frame $\mathbf{k}=\left(k,0,0\right)$ for convenience, we obtain from (23):

$$\frac{\partial\rho_k}{\partial t}=-i\rho_0 k\upsilon_{xk}\,,\quad \frac{\partial c_k}{\partial t}=-\gamma_{\rho}k^2\rho_k-\gamma_c k^2 c_k-\gamma_T k^2 T_k\,,$$
$$\frac{\partial T_k}{\partial t}=-\alpha_{\rho}k^2\rho_k-\alpha_c k^2 c_k-\alpha_T k^2 T_k-\frac{2}{3}Tik\upsilon_{xk}\,,\quad \frac{\partial\upsilon_{yk}}{\partial t}=-\eta_{\upsilon}k^2\upsilon_{yk}\,,$$
$$\frac{\partial\upsilon_{xk}}{\partial t}=-i\beta_T kT_k-i\beta_c kc_k-i\beta_{\rho}k\rho_k-\frac{4\eta_{\upsilon}}{3}k^2\upsilon_{xk}\,,\quad \frac{\partial\upsilon_{zk}}{\partial t}=-\eta_{\upsilon}k^2\upsilon_{zk}\,. \qquad (26)$$

We seek the solution of (26) in the form $\xi_{\mu k}\left(t\right)=\xi_{\mu k}\left(0\right)e^{\lambda t}$. The general solution is a superposition of six independent so-called hydrodynamic modes. Each hydrodynamic mode describes a coherent motion of the six hydrodynamic variables ξ_{μ}. Equations (26) can be rewritten in the matrix form

$$\frac{\partial\xi}{\partial t}=M\xi \qquad (27)$$

where ξ is the column of $\xi_{\mu k}$. The dispersion laws $\lambda\left(k\right)$ for the hydrodynamic modes are found from the equation

$$\det\left|M-\lambda I\right|=0\,. \qquad (28)$$

We seek the solution of (28) in the perturbation theory in the small wave vector k (k is small because the gradients (4) are small). The perturbation theory in σ is additionally used. Using (24) and the explicit expressions for $g_{e0}^{N_e(0)}$, $g_{e1}^{N_e(0)}$, $g_{e0}^{T(0)}$, $g_{e1}^{T(0)}$, $g_{i0}^{\upsilon(1)}$, we obtain the explicit expressions for the dispersion laws from (28):

$$\lambda_{1,2}=-\frac{5}{4\sqrt{2}z^4}\Lambda\sigma k^2+O\left(\sigma^2k^2,k^3\right),$$
$$\lambda_{3,4}=-\frac{3}{10}\Lambda k^2\,\frac{A\left(z\right)\pm\sqrt{A^2\left(z\right)-B\left(z\right)}}{\left(z+\sqrt{2}\right)\left(z+1\right)}+O\left(\sigma k^2,k^3\right),$$
$$\lambda_{5,6}=\pm ik\sigma\sqrt{\frac{5T\left(z+1\right)}{3m_e}}-\Lambda k^2\,\frac{5\left(29z+4\sqrt{2}\right)}{2^5z\left(z+1\right)\left(z+\sqrt{2}\right)}+O\left(\sigma^2k,\sigma k^2,k^3\right) \qquad (29)$$

where

$$A\left(z\right)=\frac{25}{8}+\frac{5}{16z^2}\left(4\sqrt{2}+13z\right),\quad B\left(z\right)=\frac{125}{8z^2}\left(\sqrt{2}+z\right),\quad \Lambda=\frac{T^{5/2}}{n_i e^4 L\sqrt{2\pi m_e}}\,, \qquad (30)$$

e is the elementary electric charge and L is the Coulomb logarithm.

Thus, the hydrodynamic modes of the Landau equation (29) are obtained in the absence of relaxation. Here the modes $\lambda_{1,2}$ describe the evolution of the transversal components of the velocity υ_n, the modes $\lambda_{3,4}$ are the diffusion and heat modes, and $\lambda_{5,6}$ are the sound modes of the system. The modes are obtained up to the second order in the small wave vector with additional account for the smallness of σ.

6. Conclusions

Hydrodynamics of completely ionized two-component plasma is investigated on the basis of the well-known Landau equation. The non-homogenous case with the finished relaxation of the component temperature and velocity is investigated. The consideration is based on the idea of the Bogolyubov functional hypothesis. This paper is concerned with the investigation of the hydrodynamic modes of the system.

The dispersion laws of the hydrodynamic modes of the system are obtained in the small wave vector perturbation theory with additional account for the small electron-to-ion mass ratio.

The results for the plasma modes described by the Vlasov kinetic equation are well-known (plasma oscillations). These results cannot be obtained by our approach, which is based on the Landau kinetic equation. The reason is that the Landau equation does not involve a self-consistent field. But, in contrast to the Vlasov kinetic equation, it takes into account the collision integral. Thus, although the problem under consideration is a model one, our results are rather important because they describe the effect of the Landau collision integral on the modes of the system.

The results of the paper are obtained for the case where the relaxation of the component temperature and velocity is finished; nevertheless they are also important for the investigation of the relaxation in the vicinity of the hydrodynamic state [8] as they are the results of the principal order of a perturbation theory in small relaxation parameters.

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