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TUNNELLING OF RECTANGULAR WAVE PACKAGES THROUGH RESONANT QUANTUM SYSTEMS

Tunneling of wave packages through resonant quantum systems represented in the S-matrix form is investigated. An arbitrary incident wave package can be written as the superposition of Gaussian pulses and for the latter ones the exact analytic solution was obtained already. As a practically interesting case, the passage of the rectangular wave package through system with a one-resonant level is considered. Using the developed recently modified saddle-point method the passage of the wave packages of the Gaussian and rectangular forms is investigated for such a system. The time delay (Δt), argument and amplitude of the outgoing wave are calculated. The dependences of Δt tunneling time on the width and energy of resonance levels, as well as on the momentum of the incident wave-package are presented. The results are compared with the ones obtained for tunneling of the one Gaussian package. Possible applications are discussed.

Keywords: quantum system, tunneling, time delay, resonance.

У роботі досліджується туннелювання хвильових пакетів крізь відкриті квантові системи з резонансними рівнями, представлені в S-матричному вигляді. Пакет довільної форми, що подається на вхід квантової системи, можна записати у вигляді суперпозиції пакетів гаусівської форми, а для останніх уже було отримано точний аналітичний розв'язок. Цікавим з практичної точки зору є дослідження проходження пакету прямокутної форми крізь квантову систему з одним резонансним рівнем. Використовуючи розроблений раніше модифікований метод сідлової точки, розглядається проходження пакетів гаусівської та прямокутної форми для такої системи. Розраховуються час затримки (Δt), аргумент і амплітуда пакету, що виходить. Наводяться графіки залежності Δt від ширини й енергії резонансних рівнів системи, а також імпульсу хвильового пакету, що подається на вхід квантової системи . Отримані результати порівнюються з результатами тунелювання одного гаусівського пакету. Обговорюються можливі застосування отриманих результатів.

Ключові слова: квантові системи, тунелювання, час затримки, резонанс.

В работе исследуется туннелирование волновых пакетов через открытые резонансные квантовые системы, представленные в S-матричном виде. Пакет произвольной формы, подаваемый на вход квантовой системы можно записать в виде суперпозиции пакетов гауссовой формы, а для них ранее уже получено точное аналитическое решение. Интересным с практической точки зрения является исследование прохождения пакета прямоугольной формы через квантовую систему с одним резонансным уровнем. Используя разработанный ранее модифицированный метод седловой точки, рассматривается прохождение пакетов гауссовой и прямоугольной формы в такой системе. Рассчитываются время задержки Δt, аргумент и амплитуда выходящего пакета. Приводятся графики зависимости Δt от ширины и энергии резонансных уровней системы, а также исходного импульса волнового пакета. Полученные результаты сравниваются с результатами туннелирования пакета гауссовой формы. Обсуждаются возможные применения полученных результатов.

Ключевые слова: квантовые системы, туннелирование, время задержки, резонанс.

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1. Introduction

Tunnelling of wave packages through quantum systems (QS's) with resonant levels is an interesting phenomenon that is important nowadays from practical points of view. Its description requires taking into account numerous quantum and boundary effects related with interference of incident and reflected waves. These effects are observed at pulse passage through double-barrier diodes, quantum tunnelling transistors and open heterostructures. Such type systems are widely used in modern micro- and nanoelectronics. Numerous publications are devoted to these problems (see, for instance, [1–3] and references therein). The main goals of interest are the description of the tunnelling process and time delay inside a system. Because of complication of QS's and variety of incident pulses, most investigations deal with a given QS and specified wave packages. In the literature, no general analytic approaches have been developed. However, this is of interest because just in analytic investigation the time delay and other parameters of the passage can be determined. Recently, in papers [5–7], a general solution of the scattering problem has been obtained in the analytic form for a Gaussian incident wave package. The passage amplitude was written in terms of Erfc(z) special function. In this approach, QS is presented as an S-matrix with resonant energy levels E_n and corresponding widths Γ_n. It can be expressed in terms of the incident package parameter - the packages width a. As a consequence, all the packages have a unite width and can be investigated on the equal footing. Then a generalized saddle-point method can be used to calculate the passage amplitude. After that, one can return back to initial parameters in the problem. This opens a possibility for investigations of packages of other form by using a superposition principle.

In the present paper, we investigate the tunnelling of the rectangular wave package through QS with a one-resonant level that has a practical interest. We present the incident pulse as three Gaussian ones with close momenta and widths. The passage amplitude is obtained as the superposition of each considered mode. We calculate the main parameters of the outgoing amplitude, in particular, time delay and compare the results with that for one Gaussian mode passing. The comparison shows that the outgoing rectangular wave packet has parameters which are more attractive for various applications. The paper is organized as follows. In the next section we give necessary information on the description of QS's and the applied approach. In Sec. 3 we present the obtained results and make a comparison of passages for considered packages. Other possible applications are also discussed.

2. Rectangular packages

Tunnelling of rectangular wave packages is of interest because pulses of such type can be easily generated in laboratories. To relate the ingoing states before scattering with outgoing states after interaction, the S-matrix formalism of scattering theory is widely used. The S-matrix elements can be calculated by different methods. For example, the expansion of scattering amplitude in perturbation theory or investigation of its general properties can be used. In what follows, we apply the S-matrix for describing the passage of the wave packages through QS. According to the general procedure [5, 6], we introduce dimensionless variables for describing both the incident wave package and the QS. The S-matrix of QS is given as a superposition of the resonant level k_j of the type

$$\frac{a_j}{k - k_j + \frac{i\Gamma_j}{2}}.$$

Here, $k_j = \frac{\sqrt{2mE_j}}{\hbar}$ corresponds to the energy of the level and Γ_j is the level width. Then the outgoing wave package is related with the incident one as follows

$$\Psi = \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \Psi_0(k) S(k) e^{(ikx - \frac{i\hbar}{4m\pi} k^2 t)} dk \, . \tag{1}$$

The incident rectangular package (Fig. 1) can be presented as the superposition of three Gaussian's pulses. As it is seen from Fig. 1, a few Gaussian pulses are sufficient to realize a packet of such type quite well. The number of the constituents included into superposition does not influence calculation procedure but considerably increases needed computational time. For example, the computer time for four constituents taken in the superposition is two orders larger than the time for three terms in the sum. So, to optimize calculations, we present the incident state as the superposition of three Gaussian pulses

$$\Psi = \sum_{i=1}^{3} A_i e^{-\frac{(s-a_i)^2}{2}} \tag{2}$$

where the coefficients A_i are chosen from the requirement of obtaining the pulse with the given width. To calculate the integral in Eq. (1), the modified saddle-point method is used [5]. It includes writing down the resonant poles entering $S(k)$ in the form

$$(k - k_j + \frac{i\Gamma_j}{2})^{-1} = e^{-\ln(k - k_j + \frac{i\Gamma_j}{2})} \tag{3}$$

as a necessary step and calculating the saddle points of the expression obtained in the exponential. In such a way complicated relations between different parameters of the QS and the package can be accounted for exactly and the regular part of the amplitude can be obtained as an asymptotic expansion.

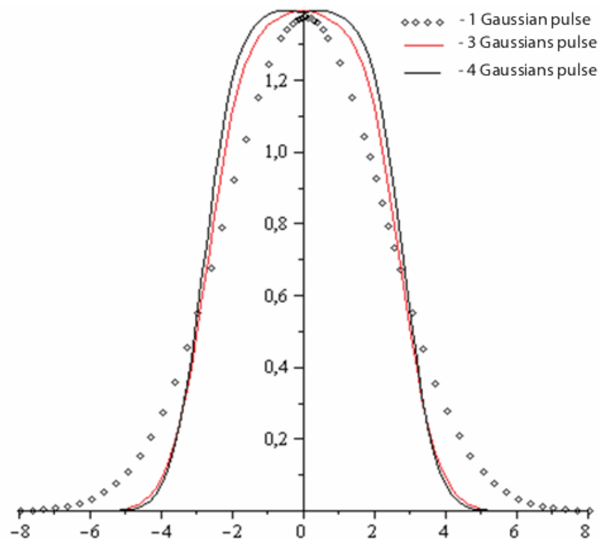

Fig. 1. The form of incident wave package tunneling through the quantum system

The outgoing wave package is given by the expression

$$\Psi(q>0,\tau) \sim \frac{1}{a} e^{il_0(q - \frac{1}{2} l_0 \tau)} \left(\sum_i e^{g(s_i)} \sqrt{-\frac{1}{e^{g''(s_i)}} f(s_i)} \right) \tag{4}$$

where s_i are saddle-points.

The wave function of the outgoing package differs from the incident wave package by some phase factor. The factor depends on the kinetic energy of the incident state, the potential energy of the barrier and time delay. The time delay can be calculated from the equation (see, for instance, [4])

$$\Delta t = \frac{dArg(\Psi)}{dE}. \tag{5}$$

Hence, the advantage of the analytic methods is obvious.

The considered incident package is shown in Fig. 1 and the outgoing state in Fig. 2. The values of the coefficients are in the subscriptions for the figures. The time delay depends on the package and QS parameters. Some illustrations are shown in Figs. 3, 4, 5.

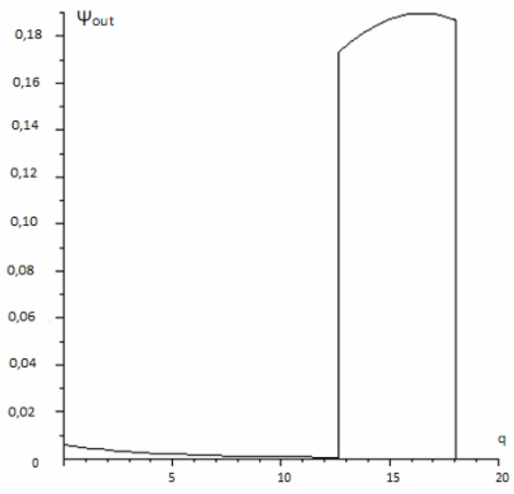

Fig. 2. The outgoing wave package form for the rectangular package tunneling with the parameters Γ_0=0.02, τ=300, k_1=1, k_0=6.

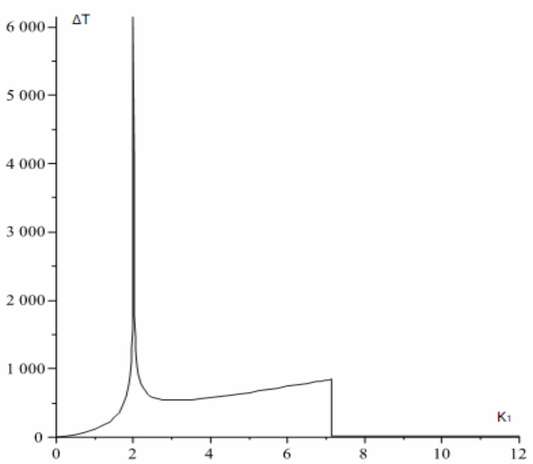

Fig. 3. The time delay from the S-matrix poles for the rectangular package tunnelling with the parameters Γ_0=0.02, τ=300, k_1=1, k_0=6.

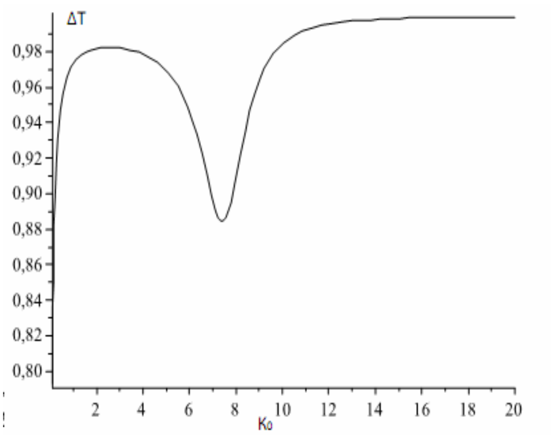

Fig. 4. The time delay of the rectangular package tunnelling with the parameters Γ_0 = 0.02, τ = 300, k_1= 1, q = 30

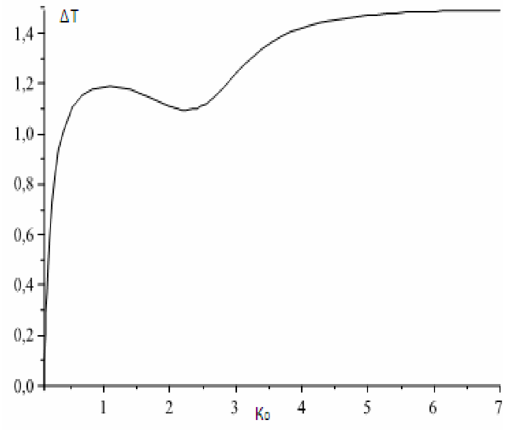

Fig. 5. The time delay of the Gaussian package tunnelling with the parameters Γ_0 = 0.02, τ = 300, k_1 = 1, q=30

3. Conclusions

In the present paper, we make use of advantages of the approach to investigating the pulse propagation through resonant QS's, which is based on the modified saddle-point method and S-matrix formalism. It gives a possibility for investigation of the tunnelling process. The Gaussian package tunnelling is an exactly solvable problem. By representing an incident wave package as the superposition of Gaussian packets, one can obtain exact analytical solutions for a wide class of problems which are interesting in practice. In the present paper, the case of the rectangular package represented as a superposition of three Gaussian pulses was investigated in detail and the results are compared with the ones for the Gaussian pulse with the same characteristics. As we can see from Figs. 2, 6, the former package demonstrates much better stability than the Gaussian one. Really, the center of mass of the rectangular package is observed at a distance twice longer than the center of mass for the Gaussian pulse. It means that the velocity of the first packet exceeds that of the latter one approximately twice. The same can be seen from Figs. 4 and 5. Really, for the incident momentum $k_0 = 6$ the time delay for the Gaussian pulse is almost twice larger than that for the rectangular package. Hence, we can conclude that the usage of rectangular packages in information transmission has to be more advantageous. This is because a large tunnelling velocity makes them more profitable in microelectronics units. Moreover, a stable form of rectangular packages makes them more easily recognizable by electronics devices.

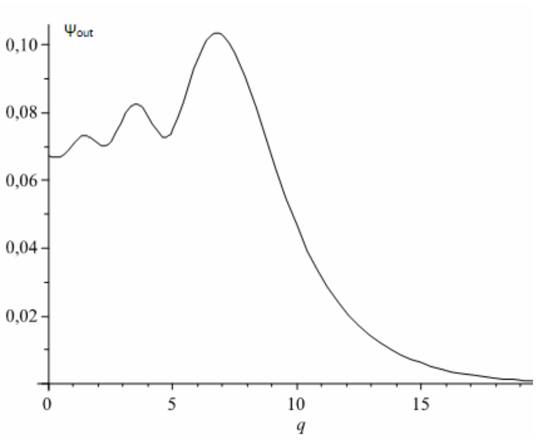

Fig. 6. The outgoing wave package form for the Gaussian package tunnelling with the parameters $\Gamma_0 = 0.02$, $\tau = 300$, $k_1 = 1$, $k_0 = 6$.

As a conclusion, we note that the problem of the resonant transmission of pulses of a general type through QS's can be investigated analytically. The results obtained here can find various applications in microelectronics. In particular, due to explicit expressions and their simplicity, it is possible to work out a universal mathematical packet allowing to calculate automatically all the parameters of the transmitted pulse for various QS's and incident packages of interest.

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