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WAVE DIFFRACTION BY SEMI-INFINITE PERIODICAL MULTI-ELEMENT KNIFE-TYPE STRIP GRATING OF FRACTALS

The diffraction problem for the H-polarized wave by a semi-infinite knife-type grating which consists of fractals is considered. The grating is the semi-infinite system of identical layers. Every layer is a finite-element plane fractal strip grating based on the pre-Cantor set in turn. The operator method is used for solving the problem. Nonlinear operator equation relatively an unknown reflection operator is obtained. The regularization procedure which connected with the elimination of singularities is carried out. The scattered field can be represented as a superposition of field of cylindrical waves which is originated as a result of scattering by the semi-infinite grating end and plane waves corresponding to infinite periodical grating. The magnitude of the reflection coefficient of plane waves of the gratings under consideration and directional patterns of cylindrical waves are presented.

Keywords: semi-infinite grating, operator method, successive over-relaxation method, regularization procedure.

Розглянуто задачу дифракції H-поляризованої хвилі на напівнескінченній періодичній ножовій решітці з фракталів. Решітка являє собою напівнескінченну систему однакових шарів. Кожен шар, у свою чергу, є скінченноелементна плоска фрактальна стрічкова решітка на основі передканторової множини. Для розв'язання застосовується операторний метод. Отримано нелінійне операторне рівняння відносно невідомого оператора відбиття. Проведено процедуру його регуляризації, яка пов'язана з виключенням особливостей. Розсіяне поле можна представити у вигляді суперпозиції полів циліндричних хвиль, створених краєм напівнескінченної структури та плоских хвиль, що відповідають нескінченній періодичній решітці. Наведені значення коефіцієнта відбиття плоских хвиль розглянутих решіток та діаграми спрямованості циліндричних хвиль.

Ключові слова: напівнескінченна решітка, операторний метод, метод релаксації, процедура регуляризації.

Рассматривается задача дифракции H-поляризованной волны на полубесконечной периодической ножевой решетке из фракталов. Решетка представляет собой полубесконечную систему одинаковых слоев. Каждый слой, в свою очередь, является конечноэлементной плоской ленточной фрактальной решеткой на основе предканторового множества. Для решения применяется операторный метод. Получено нелинейное операторное уравнение относительно неизвестного оператора отражения. Проведена процедура его регуляризации, связанная с исключением особенностей. Рассеянное поле можно представить в виде суперпозиции полей цилиндрических волн, созданных краем полубесконечной структуры и плоских волн, соответствующих бесконечной периодической решетке. Представлены значения коэффициента отражения плоских волн рассматриваемых решеток и диаграммы направленности цилиндрических волн.

Ключевые слова: полубесконечная решетка, операторный метод, метод релаксации, процедура регуляризации.

1. Introduction

Knife-type gratings are widely used for designing polarizators, frequency-selective devices, reflection screens, etc. [1-3]. Among plane diffraction gratings we should mention special type of gratings, namely, the gratings based on fractals. Such gratings s are attractive because of their compact size and their multiband property. The fractal gratings have application in medicine, military technology and cellular systems [4].

It is interesting to study a semi-infinite periodical structure which consisting of fractal gratings. The model of semi-infinite structure allows describing the field which is reflected from the end of a real finite grating.

In a large number of papers the semi-finite gratings which consist of cylindrical scatters are considered. In [5-9] the Wiener-Hopf method was used in supposition of small cylinder radius and large period as compared to the wavelength. In [10] the Foldy method is used in the case of Dirichlet boundary conditions. It is supposed that the transverse size of the scatters is small as compared to the wavelength. In [11] the method proposed in [10] is developed for the semi-infinite grating of circular cylinder scatters of arbitrary radius both in the case of Dirichlet and Neumann boundary conditions.

The semi-infinite system of grooves in a perfectly conducting plane is considered in [12] using an overlapping T-block method. The full field is represented as a sum of fields scattered by each groove. At a sufficient large distance from the end of the semi-infinite grating, the coefficients of a sum are replaced by corresponding coefficients of infinite periodical structure.

In [13-16] the semi-infinite periodical gratings with a single strip on period are considered. As a rule, diffraction problems for such gratings are solved in assumption of a single mode current distribution on the strips. The current on the strips are represented as a sum of current corresponding to the infinite grating and contribution induced by the end of the grating. The semi-infinite strip grating with small period as compared to the wavelength is considered in [16]. Using the approximate boundary conditions method the problem is reduced to the canonical one which is solved with the use of the Sommerfeld-Maliuzhinets method.

The strict solution of the diffraction problem by different semi-infinite periodical systems of obstacles is obtained by the operator method [17-23]. The reflected field is expressed via the reflection operator which is obtained from nonlinear operator equations.

In all papers mentioned above single-element semi-infinite gratings are considered, i.e. gratings with just a single element on period (strip or cylinder) and the gratings consist of these single elements placed periodically. In this paper the semi-infinite multi-element periodical grating will be studied by the operator method. As a single element we choose multi-element strip gratings based on the pre-Cantor set (fractal gratings). Single elements are placed in parallel planes one under another forming so-called knife-type grating.

2. The problem statement

Let us place the first multi-element grating in the plane $z=0$ so that its center coincides with the y- axis. Let us place every next $(n+1)$th grating in the plane $z=-nh$ so that strips were one under another, and denote the distance between ends of a single-layer grating or its width as $2d$. The structure geometry is presented in Fig.1. The time dependence is assumed to be $e^{-i\omega t}$. Suppose that from the half-space $z>0$ the H - polarized plane wave with spectral function (Fourier amplitude) $q(\xi)$ incidents on the formed multi-element knife-type grating. Then single non-zero magnetic component of the incident field may be represented in the form

$$H_x^i(y,z) = \int_{-\infty}^{\infty} q(\zeta)\exp(ik\zeta y - ik\gamma(\zeta)z)d\zeta \ ,$$

where $\gamma(\zeta) = \sqrt{1-\zeta^2}$, $\operatorname{Re}\gamma \geq 0$, $\operatorname{Im}\gamma \geq 0$, $k = 2\pi/\lambda$ is the wavenumber. We denote the Fourier amplitudes of the reflected field and the field between the layers as $a(\xi)$, $C_n(\xi)$ and $B_n(\xi)$. Then the reflected from the semi-infinite grating field may be represented in the form

$$H_x^r(y,z) = \int_{-\infty}^{\infty} a(\xi)\exp(ik\xi y + ik\gamma(\xi)z)d\xi \ , \quad (1)$$

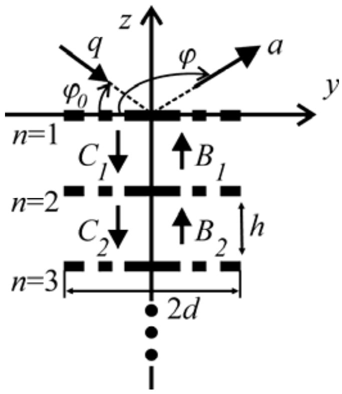

Fig. 1. Structures geometry.

$$H_x^t(y,z) = \int_{-\infty}^{\infty} C_n(\xi)\exp\big(ik\xi y - ik\gamma(\xi)(z+(n-1)h)\big)d\xi$$

$$+\int_{-\infty}^{\infty} B_n(\xi)\exp\big(ik\xi y + ik\gamma(\xi)(z+nh)\big)d\xi, \ (n-1)h < z < nh.$$

Let us introduce an integral reflection operator $\hat{\mathrm{R}}$ from the semi-infinite structure with the kernel function $\hat{R}(\xi,\zeta)$. Then the Fourier amplitude of the reflected field and (1) may be written in the form

$$a(\xi) = \int_{-\infty}^{\infty} \hat{R}(\xi,\zeta)q(\zeta)d\zeta \ , \tag{2}$$

$$H_x^{refl}(y,z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \hat{R}(\xi,\zeta)q(\zeta)\exp(ik\xi y + ik\gamma(\xi)z)d\zeta d\xi \ .$$

When we use the operator method we should know the reflection and transmission operators of a single obstacle. Suppose that we know reflection and transmission operators r and t of a single multi-element grating which may be obtained by the method of singular integral equations [24-26]. Then the Fourier amplitudes of the reflected and transmitted fields may be obtained as follows

$$(\mathrm{r}q)(\xi) = \int_{-\infty}^{\infty} r(\xi,\zeta)q(\zeta)d\zeta \ , \qquad (\mathrm{t}q)(\xi) = \int_{-\infty}^{\infty} t(\xi,\zeta)q(\zeta)d\zeta \ ,$$

and

$$t(\xi,\zeta) = \delta(\xi-\zeta) - r(\xi,\zeta) \tag{3}$$

where $\delta(\xi)$ is the Dirac delta function.

3. Operator equations

The Fourier amplitudes of the reflected field and the field between layers are connected by the following operator equations [18], [23]

$$a = \mathrm{r}q + \mathrm{te}B_1 , \tag{4}$$

$$C_1 = \mathrm{t}q + \mathrm{re}B_1 , \tag{5}$$

$$B_1 = \hat{\mathrm{R}}\mathrm{e}C_1 , \tag{6}$$

$$B_n = \hat{\mathrm{R}}\mathrm{e}C_n, \tag{7}$$
$$C_n = \mathrm{t}_{n-1}q + \mathrm{r}_n\mathrm{e}B_n,\ n = 2,3,...$$
where operator e determines the amplitudes variation of the field when the coordinate system is shifted by the distance h along the direction of the field propagation
$$(\mathrm{e}q)(\xi) = \exp(ikh\gamma(\xi))q(\xi).$$

Notice that along with (4) the Fourier amplitude of the reflected field may be expressed as follows (2)
$$a = \hat{\mathrm{R}}q. \tag{8}$$

Then after substitution (5) into (6), and (8) into (4), and using (3) we may obtain the following operator equation relatively the unknown reflection operator $\hat{\mathrm{R}}$:
$$\hat{\mathrm{R}}q = \mathrm{r}q + \mathrm{e}B_1 - \mathrm{re}B_1, \tag{9}$$
$$B_1 = \hat{\mathrm{R}}\mathrm{e}q - \hat{\mathrm{R}}\mathrm{er}q + \hat{\mathrm{R}}\mathrm{ere}B_1. \tag{10}$$

Since the scattered field may by represented as a sum of the fields with discrete and continuous spectra, then the operator $\hat{\mathrm{R}}$ may contain singularities. After substitution (10) into (9) one can see that the kernel function of the operator $\hat{\mathrm{R}}$ may have singularities in the points coinciding with the zeros of the function
$$f(\xi,\zeta) = 1 - \exp(ikh(\gamma(\xi) + \gamma(\zeta))).$$

These points correspond to the cut-off frequencies of spatial harmonics of the infinite periodical grating and they are the poles. Then the operator $\hat{\mathrm{R}}$ is the singular integral operator. For every fixed ξ denote the zeros of the function $f(\xi,\zeta)$ as
$$\zeta_l(\xi) = \mathrm{sgn}(l)\sqrt{1 - \left(\frac{2\pi|l|}{kh} - \sqrt{1-\xi^2}\right)^2},\ l = -N(\xi),...,N(\xi),$$
and for every fixed ζ denote the zeros of the function $f(\xi,\zeta)$ as
$$\xi_l(\zeta) = \mathrm{sgn}(l)\sqrt{1 - \left(\frac{2\pi|l|}{kh} - \sqrt{1-\zeta^2}\right)^2},\ l = -N(\zeta),...,N(\zeta).$$

For the elimination of singularities the regularization procedure is needed. The regularization procedure consists in the following. Such function is added to the integrand which has singularities, so that their sum does not have singularities and the integral could be calculated with the use of the quadrature formulae, and the integral for the term which we add could be calculated analytically. To remain the identity, the same term is subtracted.

Let us introduce operator R as follows
$$\mathrm{R} = \hat{\mathrm{R}} - \mathrm{s}^-\mathrm{e}\hat{\mathrm{R}}\mathrm{es}^+ \tag{11}$$
with the kernel function
$$R(\xi,\zeta) = \hat{R}(\xi,\zeta)f(\xi,\zeta).$$

In (11) we subtract singularities from the operator $\hat{\mathrm{R}}$, so the operator R does not have singularities. Let us write the action of the operator $\hat{\mathrm{R}}$ on an arbitrary function $g(\xi)$ with the use of the regularizing operator F_1 [23]

$$(\hat{\mathrm{R}}q)(\xi) = (\mathrm{RF}_1 q)(\xi)\,. \tag{12}$$

By analogy with using the operator regularizing operator F_0 we can write

$$(\mathrm{r}\hat{\mathrm{R}})(\eta,\zeta) = (\mathrm{rF}_0\mathrm{R})(\eta,\zeta). \tag{13}$$

In the expanded form the expressions (12) and (13) may be written as follows

$$(\mathrm{RF}_1 q)(\xi) = PV\int_{-\infty}^{\infty} R(\xi,\zeta)q(\zeta)\frac{\prod_{m=-N(\zeta)}^{N(\zeta)}(\zeta-\zeta_m(\xi))}{f(\xi,\zeta)}\frac{d\zeta}{\prod_{m=-N(\zeta)}^{N(\zeta)}(\zeta-\zeta_m(\xi))}+$$

$$+\pi i\sum_{n=-N(\zeta)}^{N(\zeta)} R(\xi,\zeta_n(\xi))\alpha_n(\xi)q(\zeta_n(\xi)),$$

$$(\mathrm{rF}_0\mathrm{R})(\eta,\zeta) = PV\int_{-\infty}^{\infty} r(\eta,\xi)R(\xi,\zeta)\frac{\prod_{m=-N(\xi)}^{N(\xi)}(\xi-\xi_l(\zeta))}{f(\xi,\zeta)}\frac{d\xi}{\prod_{m=-N(\xi)}^{N(\xi)}(\xi-\xi_m(\zeta))}+$$

$$+\pi i\sum_{n=-N(\xi)}^{N(\xi)} r(\eta,\xi_n(\zeta))R(\xi_n(\zeta),\zeta)\beta_n(\zeta),$$

where $\alpha_n(\xi) = \lim\limits_{\zeta\to\zeta_n(\xi)}\dfrac{\zeta-\zeta_n(\xi)}{f(\xi,\zeta)}$, $\beta_n(\zeta) = \lim\limits_{\xi\to\xi_n(\zeta)}\dfrac{\xi-\xi_n(\zeta)}{f(\xi,\zeta)}$. During deriving these expressions the following relation is used

$$PV\int_{-\infty}^{\infty}\frac{d\zeta}{\zeta-\xi} = 0\,.$$

Then (8) and (9) may be rewritten in the form

$$\begin{aligned}\mathrm{R} &= \mathrm{r} + \mathrm{e}(\mathrm{I}-\mathrm{RFere})^{-1}(\mathrm{RFereF}_0\mathrm{Re}-\mathrm{RFer}) - \\ &- \mathrm{re}(\mathrm{I}-\mathrm{RFere})^{-1}(\mathrm{RFereF}_0\mathrm{Re}-\mathrm{RFer}) - \mathrm{reF}_0\mathrm{Re},\end{aligned} \tag{14}$$

where I is the unit operator. Eq. (14) is a nonlinear operator equation relatively the unknown operator R connected with the unknown reflection operator by (11). One may solve (14) with the use of the iterative procedure with the relaxation parameter [27]

$$\widetilde{\mathrm{R}}_j = \widetilde{\tau}^{-1}\left((\widetilde{\tau}-\mathrm{I})\widetilde{\mathrm{R}}_{j-1} + \mathrm{P}(\widetilde{\mathrm{R}}_{j-1})\right)$$

where $\widetilde{\mathrm{R}} = (\widetilde{R}^i)_{i=1}^{N\times N}$ is the vector obtained after discretization of the integral operator R from its matrix by simple renumbering its elements, P is the right side of (14), N is dimension, $\widetilde{\tau}$ is diagonal matrix with elements τ of $(N\cdot N)\times(N\cdot N)$ dimension, τ is the real parameter, $\widetilde{\mathrm{R}}_j$ is the solution obtained on the j th iteration, $j=0,1,2,\ldots$. Due to the appropriate choice of the parameter τ we managed to archive the vanishing of the error.

4. Numerical results

We suppose that plane wave with unit amplitude incidents on the grating. The angle of incidence is $\varphi_0 = 90^0$. The scattered field by the semi-infinite grating may be represented as a superposition of the field of cylindrical waves appeared as a result of scattering by the end of the grating and plane waves corresponding to an infinite periodical part of the grating. Using the saddle-point method [28] when $kr >> 1$ the scattered field may be represented in the form [23]

$$H_x^r(\varphi,r) = H_x^F(\varphi,r) + H_x^c(\varphi,r) + H_x^{erfc}(\varphi,r)\,. \tag{15}$$

The first term in (15) $H_x^F(\varphi,r)$ is a set of plane waves (Floquet's modes) and does not decrease when $kr \to \infty$. The second term in (15) $H_x^c(\varphi,r)$ is a cylindrical wave scattered by the end of the grating. Its magnitude decreases as $1/\sqrt{kr}$ when $kr \to \infty$. The third term in (15) $H_x^{erfc}(\varphi,r)$ takes into account integrand singularities and provides the uniform asymptotic representation of the field when $kr \to \infty$. It is expressed in terms of Gauss error function. Here (r,φ) is the polar coordinate system.

We introduce function which describes the field without influence of the plane waves

$$D(\varphi,r) = 10\log\left|H_x^c(\varphi,r) + H_x^{erfc}(\varphi,r)\right|^2. \tag{16}$$

Function $D(\varphi,\rho)$ is the similar to the direction pattern for the finite grating. However we should notice that due to the term $H_x^{erfc}(\varphi,r)$ sum $\left|H_x^c(\varphi,r) + H_x^{erfc}(\varphi,r)\right|$ does not decrease when r increases.

Introduce reflection coefficient of the plane waves as follows

$$RC = |\,2\pi R(w,\zeta_0)\,|\,.$$

Notice that the reflected plane wave does not exist in the total domain $\varphi \in (0,\pi)$. It is obvious that RC does not depend on the distance r.

Let us compare the characteristics of the scattered fields by the semi-infinite gratings of single layers which consist of a single strip and pre-Cantor grating. Denote as K_n the set obtained on the n th step of the creation of the Cantor set in the interval $(-d;d)$ [29]:

$$K_0 = (-d;d)\,,$$

$$K_1 = \left(-d;-d+\frac{2d}{3}\right)\cup\left(-d+\frac{2\cdot 2d}{3};d\right),$$

$$K_2 = \left(-d;-d+\frac{2d}{9}\right)\cup\left(-d+\frac{2\cdot 2d}{9};-d+\frac{2d}{3}\right)\cup\left(-d+\frac{2\cdot 2d}{3};-d+\frac{7\cdot 2d}{9}\right)$$
$$\cup\left(-d+\frac{8\cdot 2d}{9};d\right),$$

and so on. Denote $CK_n = (-d;d)\setminus K_n$ the complement of the set K_n in the interval $(-d;d)$. Let us call CK_n the pre-Cantor set of the order n.

The results in Figs. 2, 3, and 4 are presented for the same values of the distance between layers $kh = 5$ ($h/\lambda = 0.8$), but different single layer grating geometry. Introduce parameter L which is equal to the sum of the length of all strips of a single layer (along y- axis). The results presented in Fig. 2 a), Fig. 3 (curve 1) and Fig. 4 (curve 1) are obtained for a single strip with a width of half of the wavelength, $kd = kL/2 = 1.57$ ($d/\lambda = 0.25$).The reflection coefficient of the plane wave equals to $RC = 0.1279$. In Figs. 2 b), 3 (curve 2) and 4 (curve 2) the results for the pre-Cantor grating of the forth order are presented, $n = 4$, $kd = 1.57$, $kL = 2.52$. The reflection coefficient for the grating with a single layer based on CK_4 equals to $RC = 0.02528$. In Fig. 2 c), Fig. 3 (curve 3) and Fig. 4 (curve 3) the results are calculated for a single strip with a width that equals to a sum of strips width of the pre-Cantor grating, $kd = 1.26$, $kL = 2.52$. The reflection coefficient in this case equals $RC = 0.09628$. The choice of such structure parameters allows comparing semi-infinite knife-type grating with layers consisting of the pre-Cantor grating and a single strip. Notice that the reflection coefficient for the grating based on the pre-Cantor set almost 4 times smaller than the reflection coefficient of the single-element grating with the same parameter $kL = 2.52$.

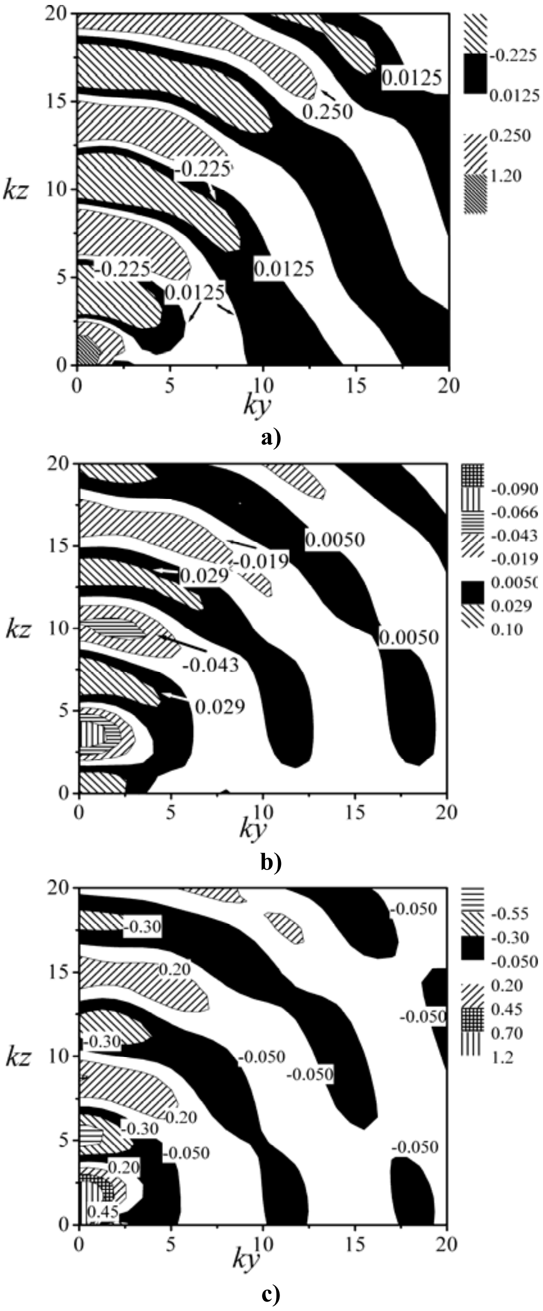

Fig. 2. Real part of the reflected field component $\mathrm{Re}(H_x^r)$.

Fig. 2 presents the reflected near-field distribution (real part of the magnetic field component H_x^r). Two type of waves propagate away from the grating. One of them is a cylindrical wave appeared as a result of scattering by the end of the grating. The field maximum of such wave situated at $\varphi = 90^0$. Another one is a plane wave that corresponds to an infinite periodical part of the grating. The structure parameters are chosen so that only one reflected plane wave can propagate in the domain $y > 0$, $z > 0$.

It is seen from the figure that the plane wave exists only in the domain $w < \varphi$ where $w \approx 165^0$ is the propagation angle of the reflected plane wave which corresponds to the periodical part of the grating.

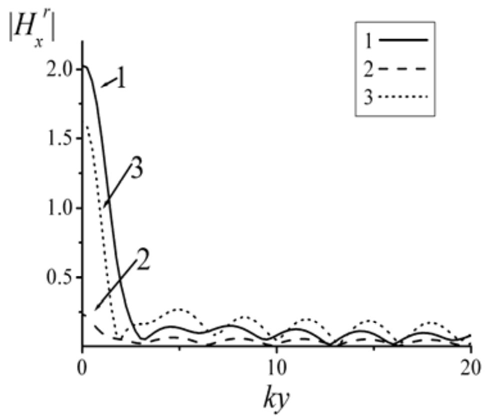

Fig. 3. Module of the reflected field component $\left|H_x^r\right|$, $z\lambda = 0.1$.

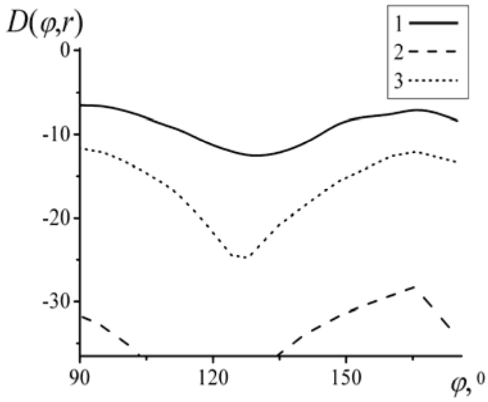

Fig. 4. Dependence of $D(\varphi, r)$ **vs.** φ.

It is suitable [22] to calculate the scattered field at the distance $z = 0.1\lambda$ from the structure (Fig. 3). The field maximum is located near $y = 0$ above the middle of a single strip. For a single strip with the decrease of its width $2d$ the field maximum decreases. The field maximum for the pre-Cantor grating is significantly smaller than for a single strip. In the domain above the grating the oscillations are present which appear as a result of influence of the evanescent plane waves on the near field.

Fig. 4 shows the dependencies of the $D(\varphi, r)$ vs. polar angle φ when $kr = 30$. One may observe the first maximum near angle $\varphi = \varphi_0 = 90^0$ in the dependencies. The second maximum near angle $\varphi = w = 165^0$ is connected with the excitation of the plane wave of the periodic part of the grating. The value of this maximum, mainly, is defined be the term $H_x^{erfc}(\varphi, r)$ in (16).

It is obvious that since the grating is symmetric relatively the z-axis, all presented dependencies are also symmetric relatively the z-axis.

From Fig. 2, Fig. 3, and Fig. 4 one can see that the amplitude of the field reflected by the pre-Cantor grating is sufficiently smaller than the amplitude of the reflected field by the single-element grating. Thus the pre-Cantor grating is practically transparent for the incident wave.

The conducted numerical study shows that the increase of the order of the pre-Cantor set $n > 4$ leads to negligible changes in characteristics of scattered fields, and in the same time it significantly complicates the manufacturing of such gratings.

5. Conclusions

In this paper the semi-infinite multi-element knife-type grating is studied numerically for the first time. As a single layer the finite plane grating based on fractal or the pre-Cantor set is chosen. The problem is reduced to the nonlinear operator equation. After its regularization which is connected with the poles exclusion, it is replaced by the matrix one. For solving it the iterative procedure with the relaxation parameter is proposed.

The scattered far and near fields are studied. It is established that the geometry of a strips displacement in a single layer has a greater influence on the field amplitude than their width. So the reflected field amplitude by the pre-Cantor grating is sufficiently small compared to the reflected field amplitude by the single-element one. The choice of the pre-Cantor grating of the 4th order ($n = 4$) is optimal and the increase of the order n does not introduce significant changes in the characteristics of the scattered fields.

Thus multi-layer periodical gratings based on pre-Cantor set are perspective for designing of radiolucent and radiotransparent covers, antennas systems and other functional microwave devices.

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