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ONE-VELOCITY AND ONE-TEMPERATURE HYDRODYNAMICS OF PLASMA

The hydrodynamics of fully ionized plasma where the relaxation of the component temperatures and velocities is finished is investigated on the basis of the Landau kinetic equation. The reduced description parameters of the system are the component particle densities, the macroscopic velocity and the temperature of the system. The hydrodynamics is built starting from the Bogolyubov functional hypothesis. The consideration is based on a perturbation theory in small gradients of the reduced description parameters. The component distribution functions are found in the perturbation theory with accuracy up to the first order in the gradients. Hydrodynamic equations for the reduced description parameters are built taking into account dissipative processes. The obtained integral equations are solved by expansion in the Sonine polynomial series with the additional use of the electron-to-ion mass ratio smallness. The kinetic coefficients of the system are calculated taking into account smallness of the mass ratio. These results of the work are not only important themselves, but can be a basis for the investigation of relaxation phenomena at their final stage as a main approximation. The obtained hydrodynamic equations can be used for the hydrodynamic mode investigation.

Keywords: fully ionized election-ion plasma, Landau kinetic equation, distribution functions, hydrodynamic equations, kinetic coefficients.

На основе кинетического уравнения Ландау изучается гидродинамика полностью ионизированной плазмы в случае завершенной релаксации скоростей и температур компонент. Параметрами сокращенного описания системы являются плотности числа частиц компонент, макроскопическая скорость и температура системы. Гидродинамика строится исходя из функциональной гипотезы Боголюбова. Рассмотрение базируется на теории возмущений по малым градиентам параметров сокращенного описания. Функции распределения компонент ищутся в теории возмущений с точностью до членов первого порядка по градиентам. Построены уравнения гидродинамики для параметров сокращенного описания с учетом диссипативных процессов. Полученные интегральные уравнения решаются разложением в ряд по полиномам Сонина с дополнительным учетом малости отношения масс электрона и иона. Кинетические коэффициенты системы вычислены с учетом малости отношения масс. Результаты работы не только важны сами по себе, но также могут быть основой для исследования релаксационных явлений вблизи завершения релаксации как главное приближение. Полученные уравнения гидродинамики могут быть использованы для изучения гидродинамических мод системы.

Ключевые слова: полностью ионизированная электрон-ионная плазма, кинетическое уравнение Ландау, функции распределения, уравнения гидродинамики, кинетические коэффициенты.

На основі кінетичного рівняння Ландау вивчається гідродинаміка повністю іонізованої плазми у випадку завершеної релаксації швидкостей та температур компонент. Параметрами скороченого опису системи є густини кількості частинок компонент, макроскопічна швидкість та температура системи. Гідродинаміка будується виходячи з функціональної гіпотези Боголюбова. Розгляд базується на теорії збурень за малими градієнтами параметрів скороченого опису. Функції розподілу компонент шукаються у теорії збурень з точністю до членів першого порядку по градієнтам. Побудовано рівняння гідродинаміки для параметрів скороченого опису з урахуванням дисипативних процесів. Отримані інтегральні рівняння розв'язуються розвиненням в ряд по поліномам Соніна з додатковим урахуванням малості відношення мас електрона та іона. Кінетичні коефіцієнти системи пораховано з урахуванням малості відношення мас. Результати роботи важливі не лише самі по собі, але також можуть бути використані для дослідження релаксаційних явищ поблизу завершення релаксації як головне наближення. Отримані рівняння гідродинаміки можуть бути використані для отримання гідродинамічних мод системи.

Ключові слова: повністю іонізована електрон-іонна плазма, кінетичне рівняння Ландау, функції розподілу, рівняння гідродинаміки, кінетичні коефіцієнти.

Introduction

In his famous work [1] Landau derived a kinetic equation for completely ionized gas with Coulomb interaction, which is widely used in the kinetic theory of plasma. On the basis of this equation he also studied the temperature relaxation in plasma. The problem of the relaxation times in the spatial uniform case was investigated by many authors (see, for example [2-4]). Present work is concerned with the non-homogenous case where the relaxation of the component temperatures and velocities is finished. The problem of the one-velocity and one-temperature hydrodynamics of two-component systems (a usual hydrodynamics) is not new [5-7], but it was usually investigated on the basis of the Boltzmann equation for uncharged particles.

The aim of the present work is to build usual hydrodynamics of plasma on the basis of the Landau kinetic equation and to obtain the component distribution functions and kinetic coefficients of the system. The mentioned results, besides being important themselves, obviously can be considered as the leading order approximation in the case of small differences of the component velocities and temperatures.

The article is organized as follows. First, the Landau kinetic equation is written. Then reduced description parameters of the system are introduced, and the component distribution functions are obtained in the homogenous case. Then hydrodynamic equations for the reduced description parameters are built, and the distribution functions in the non-homogenous case are calculated. Using these distributions, the kinetic coefficients of the system are found.

Basic equations of the theory

The well-known Landau kinetic equation for fully ionized electron-ion plasma is written

$$\frac{\partial f_{ap}(x,t)}{\partial t} = -\frac{p_{an}}{m_a}\frac{\partial f_{ap}(x,t)}{\partial x_n} + I_{ap}\left(f(x,t)\right),$$

$$I_{ap}\left(f\right) = -\sum_b \frac{\partial}{\partial p_n}\left[2\pi\left(e_a e_b\right)^2 L\int\left\{f_{ap}\frac{\partial f_{bp'}}{\partial p'_l} - f_{bp'}\frac{\partial f_{ap}}{\partial p_l}\right\}D_{nl}\left(\frac{p}{m_a}-\frac{p'}{m_b}\right)d^3p'\right] \tag{1}$$

where

$$D_{nl}\left(u\right) \equiv (|u|^2\,\delta_{nl} - u_n u_l)/|u|^2 \tag{2}$$

Here $f_{ap}(x,t)$ is distribution function of the a-th component of the plasma (a, b, c,… $=e,i$). It is normalized by relation

$$\int f_{ap}(x,t)d^3p = n_a(x,t) \tag{3}$$

where $n_a(x,t)$ is number of particle number density of the a-th component. The Landau equation is a model one but it adequately describes the role of the Coulomb interaction in the system at long distances. Therefore, it is widely used in the plasma theory.

As is known [5], the reduced description parameters in the one-velocity and one-temperature hydrodynamics are the particle number densities of the components $n_a(\vec{x},t)$, the temperature $T(x,t)$ and the velocity $\upsilon_n(x,t)$ of the system. By definition, these parameters are introduced as follows:

$$\pi_n = \sum_a \int f_{ap}p_n d^3p = \upsilon_n\rho \qquad (\rho \equiv n_e m_e + n_i m_i),$$

$$\varepsilon = \sum_a \int f_{ap}\varepsilon_{ap}d^3p = \frac{3}{2}nT + \frac{1}{2}\rho\upsilon^2 \qquad (n \equiv n_e + n_i) \tag{4}$$

where π_n and ε are the total momentum and the energy densities of the system, respectively ($\varepsilon_{ap} \equiv p^2/2m_a$).

The investigation is based on the functional hypothesis [5], which can be written as

$$f_{ap}\left(x,t\right) \xrightarrow[t\gg\tau_0]{} f_{ap}\left(x,\xi(t)\right) \tag{5}$$

where the reduced description parameters are denoted as ξ_μ: $\xi_0 = T$, $\xi_n = \upsilon_n$, $\xi_a = n_a$ $(\mu = 0, n, a)$. In (5) τ_0 is a time which is much longer than the subsystem velocity and temperature relaxation times. The dependence of the reduced description parameters on the coordinates is supposed to be weak, so the gradients of the reduced description parameters are assumed to be small,

$$\frac{\partial^s \xi_\mu(\vec{x})}{\partial x_{n_1} \dots \partial x_{n_s}} \sim g^s \qquad (g \ll 1). \tag{6}$$

Parameter g is estimated as $g = l_{\mathrm{f}}/L$ where l_{f} is the mean free path, L is characteristic length of inhomogeneities in the system. In what follows, the contribution of the order g^s to a quantity A is denoted by $A^{(s)}$.

According to the functional hypothesis (5), hydrodynamic equations have the structure

$$\frac{\partial \xi_\mu(x,t)}{\partial t} \equiv L_\mu(x, f(\xi(t))) \tag{7}$$

where functional $L_\mu\left(x, f\right)$ can be found from the kinetic equation (1) and definitions (4). Then equation (1) is rewritten in view of (5) and (6) as the equation for the functional $f\left(x,\xi\right)$

$$\sum_\mu \int d^3x' \frac{\delta f_{ap}\left(x,\xi\right)}{\delta \xi_\mu\left(x'\right)} L_\mu\left(x', f\left(\xi\right)\right) = -\frac{p_n}{m_a} \frac{\partial f_{ap}\left(x,\xi\right)}{\partial x_n} + I_{ap}\left(f(x,\xi)\right). \tag{8}$$

This equation should be solved with additional conditions

$$\sum_a \int f_{ap}(x,\xi) p_n d^3p \equiv \upsilon_n(x) \sum_a m_a n_a(x), \qquad \int f_{ap}(x,\xi) d^3p = n_a(x),$$

$$\sum_a \int f_{ap}(x,\xi)\varepsilon_{ap} d^3p = \frac{3}{2} T(x) \sum_a n_a(x) + \frac{1}{2}\upsilon(x)^2 \sum_a m_a n_a(x) \tag{9}$$

that follow from definitions (3), (4).

In order to realize the reduced description method, one should calculate the functional $f\left(x,\xi\right)$ from equations (8) and (9). These equations are obviously solvable in a perturbation theory in the gradients of the reduced description parameters.

Hydrodynamic equations

Hydrodynamic equations (7) are obtained from conservation laws following from the kinetic equation (1) and the definitions (4)

$$\frac{\partial n_a}{\partial t} = -\frac{1}{m_a}\frac{\partial \pi_{al}}{\partial x_l}, \qquad \frac{\partial \pi_n}{\partial t} = -\frac{\partial t_{nl}}{\partial x_l}, \qquad \frac{\partial \varepsilon}{\partial t} = -\frac{\partial q_l}{\partial x_l}. \tag{10}$$

Here the total energy and momentum fluxes in the system q_l, t_{nl}, and momentum density of the a-th component π_{al}

$$q_l = \sum_a \int d^3p \varepsilon_{ap} \frac{p_l}{m_a} f_{ap}, \qquad t_{nl} = \sum_a \int d^3p p_n \frac{p_l}{m_a} f_{ap}, \qquad \pi_{al} = \int d^3p p_l f_{ap}. \tag{11}$$

are introduced ($\pi_n = \pi_{en} + \pi_{in}$).

To build hydrodynamic equations with taking into account dissipative processes the solution of equations (8), (9) should be found in the form of a series up to the first order in the gradients of the parameters $\xi_\mu(x)$ by using an iterative procedure

$$f_{ap}(x,\xi) = f_{ap}^{(0)} + f_{ap}^{(1)} + O(g^2)\,. \tag{12}$$

Equation (8) shows that the distribution functions in the leading approximation coincide with the Maxwell ones

$$f_{ap}^{(0)} = w_{a,p-m_a\upsilon}\,, \qquad w_{ap} \equiv \frac{n_a \beta^{3/2}}{(2\pi m_a)^{3/2}} \exp(-\beta\varepsilon_{ap}) \qquad (\beta \equiv 1/T\,) \tag{13}$$

because for collision integral (1) the relation

$$I_{ap}(w) = 0 \tag{14}$$

is true.

The fluxes in the lab reference system are connected with ones in the accompanying reference system (ARS) by relations

$$q_n = q_n^o + t_{nl}^o \upsilon_l + (\varepsilon^o + \frac{1}{2}\rho\upsilon^2)\upsilon_n\,, \qquad t_{nl} = t_{nl}^o + \rho\upsilon_n\upsilon_l\,, \qquad \pi_{an} = \pi_{an}^o + m_a n_a \upsilon_n \tag{15}$$

where quantities taken in the ARS have superscript o. From (4), (11)-(13), (15) it can be obtained that the hydrodynamic equations with contributions up to the second order in gradients are given by relations

$$\frac{\partial n_a}{\partial t} = -\frac{\partial n_a \upsilon_n}{\partial x_n} - \frac{1}{m_a}\frac{\partial \pi_{an}^{o(1)}}{\partial x_n}\,, \qquad \frac{\partial \upsilon_n}{\partial t} = -\upsilon_l \frac{\partial \upsilon_n}{\partial x_l} - \frac{1}{\rho}\frac{\partial nT}{\partial x_n} - \frac{1}{\rho}\frac{\partial t_{nl}^{o(1)}}{\partial x_l}\,,$$

$$\frac{\partial T}{\partial t} = -\upsilon_n \frac{\partial T}{\partial x_n} - \frac{2}{3}T\frac{\partial \upsilon_n}{\partial x_n} - \frac{2}{3n}\frac{\partial q_n^{o(1)}}{\partial x_n} - \frac{2}{3n} t_{nl}^{o(1)} \frac{\partial \upsilon_n}{\partial x_l} + \frac{T}{n}\sum_a \left(\frac{1}{m_a}\frac{\partial \pi_{an}^{o(1)}}{\partial x_n} \right). \tag{16}$$

Here we take into account that for the considered system

$$\varepsilon^o = \frac{3}{2}nT\,, \qquad t_{nl}^{o(0)} = nT\delta_{nl}\,, \qquad q_l^{o(0)} = 0\,, \qquad \pi_{an}^{o(0)} = 0\,, \qquad \pi_n^o = 0\,. \tag{17}$$

In equations (16) fluxes of mass $\pi_{an}^{o(1)}$, momentum $t_{nl}^{o(1)}$, energy $q_n^{o(1)}$ describe dissipative processes in the system.

According to the idea of the rotational invariance, the distribution functions of the first order in gradients in the ARS have the structure

$$f_{ap}^{(1)} = w_{a,p-m_a\upsilon} \Bigg[p_n \sum_b A_a^{N_b}(\beta\varepsilon_{ap}) \frac{\partial n_b}{\partial x_n} + p_n A_a^T(\beta\varepsilon_{ap}) \frac{\partial T}{\partial x_n} + \\ + (p_l p_n - \frac{1}{3}\delta_{nl} p^2) A_a^\upsilon(\beta\varepsilon_{ap}) \frac{\partial \upsilon_n}{\partial x_l} \Bigg]_{p \to p - m_a\upsilon}. \tag{18}$$

Scalar functions $A_a^{N_b}(\beta\varepsilon_p)$, $A_a^T(\beta\varepsilon_p)$, $A_a^\upsilon(\beta\varepsilon_p)$ can be found from integral equations

$$p_n \left\{ \frac{1}{n_a m_a}\delta_{ab} - \frac{1}{\rho} \right\} = -\sum_b \int d^3p' K_{ab}(p,p')\, p'_n A_b^{N_c}(\beta\varepsilon_{bp'})\,,$$

$$p_n \frac{\beta}{m_a} \left\{ \beta\varepsilon_{ap} - \frac{3}{2} - \frac{n}{\rho} m_a \right\} = -\sum_b \int d^3p' K_{ab}(p,p')\, p'_n A_b^T(\beta\varepsilon_{bp'})\,,$$

$$\frac{\beta}{m_a} \left\{ p_l p_n - \frac{1}{3}\delta_{nl} p^2 \right\} = -\sum_b \int d^3p' K_{ab}(p,p') (p'_l p'_n - \frac{1}{3}\delta_{ln} p'^2) A_b^\upsilon(\beta\varepsilon_{bp'})\,. \tag{19}$$

Kernel $K(p,p')$ of these equations is defined through the collision integral (1) by the formulas

$$K_{ab}\left(p,p'\right)w_{ap}=-M_{ab}\left(p,p'\right)w_{bp'},\qquad M_{ab}\left(p,p'\right)=\left.\frac{\delta I_{ap}(f)}{\delta f_{bp'}}\right|_{f_{cp''}\to w_{cp''}}. \tag{20}$$

Additional conditions (9) give the following restrictions on the solution of the integral equations (19)

$$\sum_a\int d^3p w_{ap}p^2A_a^T\left(\beta\varepsilon_{ap}\right)=0,\qquad \sum_a\int d^3p w_{ap}p^2A_a^{N_b}\left(\beta\varepsilon_{ap}\right)=0, \tag{21}$$

which provide the uniqueness of the solution.

The solution of integral equations (19) will be found in the form of the expansions in the Sonine polynomials [8]

$$A_a^{N_b}\left(\beta\varepsilon_{ap}\right)=\sum_{s=0}^{\infty}g_{as}^{N_b}S_s^{3/2}\left(\beta\varepsilon_{ap}\right),\; A_a^{T}\left(\beta\varepsilon_{ap}\right)=\sum_{s=0}^{\infty}g_{as}^{T}S_s^{3/2}\left(\beta\varepsilon_{ap}\right),$$

$$A_a^{\upsilon}\left(\beta\varepsilon_{ap}\right)=\sum_{s=0}^{\infty}g_{as}^{\upsilon}S_s^{5/2}\left(\beta\varepsilon_{ap}\right). \tag{22}$$

The Sonine polynomials $S_n^{\alpha}\left(x\right)$ are defined by the relation

$$S_n^{\alpha}\left(x\right)\equiv\frac{1}{n!}e^{x}x^{-\alpha}\frac{d^n}{dx^n}\left(e^{-x}x^{\alpha+n}\right), \tag{23}$$

and have the property of orthogonality

$$\int_0^{\infty}e^{-x}x^{\alpha}S_n^{\alpha}\left(x\right)S_{n'}^{\alpha}\left(x\right)dx=\frac{\Gamma\left(n+\alpha+1\right)}{n!}\delta_{nn'}. \tag{24}$$

The Sonine polynomials are useful in the next calculation because of the equality

$$\int d^3p w_{ap}\varepsilon_p^{\alpha-1/2}S_s^{\alpha}(\beta\varepsilon_{ap})S_{s'}^{\alpha}(\beta\varepsilon_{ap})=\frac{2n_aT^{\alpha-1/2}}{\pi^{1/2}}\frac{\Gamma(n+\alpha+1)}{s!}\delta_{ss'},$$

which follows from (24).

Distribution function (18) allows calculating the first order in gradients contributions to fluxes (dissipative fluxes) in the ARS:

$$\pi_{an}^{o(1)}=\sum_b A_{ab}\frac{\partial n_b}{\partial x_n}+B_a\frac{\partial T}{\partial x_n},\qquad q_n^{o(1)}=\sum_a C_a\frac{\partial n_a}{\partial x_n}+D\frac{\partial T}{\partial x_n},$$

$$t_{nl}^{o(1)}=-\eta\left(\frac{\partial\upsilon_n}{\partial x_l}+\frac{\partial\upsilon_l}{\partial x_n}-\frac{2}{3}\frac{\partial\upsilon_m}{\partial x_m}\delta_{nl}\right), \tag{25}$$

where kinetic coefficients

$$A_{ab}=m_an_aT\,g_{a0}^{N_b},\qquad B_a=m_an_aT\,g_{a0}^{T},$$

$$C_a=\frac{5}{2}T^2\sum_b n_b\left(g_{b0}^{N_a}-g_{b1}^{N_a}\right),\qquad D=\frac{5}{2}T^2\sum_a n_a(g_{a0}^{T}-g_{a1}^{T}),$$

$$\eta=-T^2\sum_a n_am_ag_{a0}^{\upsilon}. \tag{26}$$

are introduced. Here η is shear viscosity but for other kinetic coefficients there are several standard notations (see, for example, [5 - 7]).

As seen, the usage of the Sonine polynomials is rather convenient, because the momentum density and the momentum flux are expressed in terms of only one polynomial, and the energy flux is expressed in terms of only two polynomials.

Calculation of the kinetic coefficients

Substituting expansions (22) in integral equations (19), multiplying by the Sonine polynomials and integrating over momentum give an infinite set of equations for coefficients $g_{as}^{N_b}$, g_{as}^{T}, g_{as}^{υ}. For their approximate calculation we should artificially truncate the number of polynomials in (22) (see, for example [10]) to obtain a finite set equations for the coefficients $g_{as}^{N_b}$, g_{as}^{T}, g_{as}^{υ}. From (26) it is evident that at least $A_a^{N_b}$ and A_a^{T} should be found in the two-polynomial approximation, and A_a^{υ} – in the one-polynomial one.

From (19), (22) the following set of the truncated equations for the coefficients $g_{as}^{N_b}$ ($s=0,1$), g_{as}^{T} ($s=0,1$), g_{a0}^{υ} is obtained

$$\sum_{s'=0,1}\sum_{b} G_{as,bs'} g_{bs'}^{T} = -Y_{as}, \qquad \sum_{s'=0,1}\sum_{b} G_{es,bs'} g_{bs'}^{N_e} = -\frac{3n_i m_i T}{\rho}\delta_{s0},$$
$$\sum_{s'=0,1}\sum_{b} G_{es,bs'} g_{bs'}^{N_i} = \frac{3n_e m_e T}{\rho}\delta_{s0}, \qquad \sum_{s'=0,1}\sum_{b} G_{is,bs'} g_{bs'}^{N_i} = -\frac{3n_e m_e T}{\rho}\delta_{s0},$$
$$\sum_{s'=0,1}\sum_{b} G_{is,bs'} g_{bs'}^{N_e} = \frac{3n_i m_i T}{\rho}\delta_{s0}, \qquad \sum_{b} g_{b0}^{\upsilon} H_{a0,b0} = -10 n_a m_a T \tag{27}$$

where

$$Y_{e0} = -Y_{i0} = 3n_e n_i (m_i - m_e)/\rho, \qquad Y_{a1} = -15 n_a/2. \tag{28}$$

Equations (27) contain the matrixes $G_{ak,bn}$, $H_{ak,bn}$ given by formulas

$$G_{as,bs'} \equiv \left\{ p_n S_s^{3/2}\left(\beta\varepsilon_{ap}\right), p_n S_{s'}^{3/2}\left(\beta\varepsilon_{bp}\right)\right\}_{ab},$$
$$H_{as,bs'} = \left\{ (p_l p_n - \delta_{nl} p^2/3) S_s^{5/2}\left(\beta\varepsilon_{ap}\right), (p_l p_n - \delta_{nl} p^2/3) S_{s'}^{5/2}\left(\beta\varepsilon_{bp}\right)\right\}_{ab} \tag{29}$$

which contain integral brackets defined by the relation

$$\{g,h\}_{ab} \equiv \int d^3p\, d^3p'\, g(p)\, w_{ap} K_{ab}(p,p')\, h(p'). \tag{30}$$

Using (22), relations (21) take the form

$$\sum_a m_a n_a g_{a0}^{T} = 0, \qquad \sum_a m_a n_a g_{a0}^{N_b} = 0 \tag{31}$$

and must be used as additional conditions to equations (27).

In order to simplify the obtained results, we may take into account that the electron-to-ion mass ratio is small $\sigma \equiv \sqrt{m_e/m_i} \ll 1$. Using (1), (20), (29)-(31), we obtain the coefficients $g_{as}^{N_b}$ ($s=0,1$), g_{as}^{T} ($s=0,1$), g_{a0}^{υ} in a perturbation theory in σ

$$g_{e0}^{T} = -3\frac{\sqrt{2}n_e + 7z^2 n_i}{2^{5/2}z^2 n_i\left(\sqrt{2}n_e + z^2 n_i\right)}\lambda + O(\sigma), \quad g_{e1}^{T} = \frac{3}{2^{1/2}\left(\sqrt{2}n_e + z^2 n_i\right)}\lambda + O(\sigma),$$

$$g_{i0}^{T} = \frac{3n_e\left(\sqrt{2}n_e + 7z^2 n_i\right)}{2^{5/2}z^2 n_i^2\left(\sqrt{2}n_e + z^2 n_i\right)}\lambda\sigma^2 + O\left(\sigma^3\right),$$

$$g_{i1}^{T} = \frac{3\left(5\sqrt{2}n_e + 5z^2 n_i - 4z^2 n_e\right)}{2^4 z^4 n_i\left(\sqrt{2}n_e + z^2 n_i\right)}\lambda\sigma + O\left(\sigma^2\right),$$

$$g_{e0}^{N_e} = -\frac{3T\left(4\sqrt{2}n_e + 13z^2 n_i\right)}{2^{9/2}z^2 n_i n_e\left(z^2 n_i + \sqrt{2}n_e\right)}\lambda + O(\sigma), \quad g_{i0}^{N_e} = g_{e0}^{N_i} = -\frac{n_e}{n_i}\sigma^2 g_{e0}^{N_e},$$

$$g_{e1}^{N_e} = \frac{9T}{2^{7/2}n_e\left(z^2 n_i + \sqrt{2}n_e\right)}\lambda + O(\sigma), \quad g_{e1}^{N_i} = -\frac{n_e}{n_i}\sigma^2 g_{e1}^{N_e}, \quad g_{i0}^{N_i} = -\frac{n_e}{n_i}\sigma^2 g_{i0}^{N_e}$$

$$g_{i1}^{N_e} = -\frac{9T}{2^5 z^2 n_i\left(z^2 n_i + \sqrt{2}n_e\right)}\lambda\sigma + O\left(\sigma^2\right), \quad g_{i1}^{N_i} = -\frac{n_e}{n_i}\sigma^2 g_{i1}^{N_e},$$

$$g_{e0}^{\upsilon} = -\frac{5}{2^3\left(n_e + \sqrt{2}n_i z^2\right)}\lambda + O(\sigma), \quad g_{i0}^{\upsilon} = -\frac{5}{2^3 z^4 n_i}\lambda\sigma + O\left(\sigma^2\right) \tag{32}$$

where the notation

$$\lambda = \frac{T^{1/2}}{e^4 L\left(\pi m_e\right)^{1/2}} \tag{33}$$

is introduced. As seen from (25), (26), the obtained expressions are important for calculation of the kinetic coefficients.

According to the standart definition of kinetic coefficients for the two-component systems [6, 7], we can introduce notations

$$t_{nl}^{o(1)} = -\eta\left(\frac{\partial\upsilon_n}{\partial x_l} + \frac{\partial\upsilon_l}{\partial x_n} - \frac{2}{3}\delta_{nl}\frac{\partial\upsilon_m}{\partial x_m}\right) - \zeta\delta_{nl}\frac{\partial\upsilon_m}{\partial x_m},$$

$$\pi_{en}^{o(1)} \equiv -D_e^T\frac{\partial\ln T}{\partial x_n} - \frac{n^2 m_e m_i}{\rho}D_{ei}d_n, \quad \pi_{in}^{o(1)} \equiv -D_i^T\frac{\partial\ln T}{\partial x_n} - \frac{n^2 m_e m_i}{\rho}D_{ie}d_n,$$

$$q_n^{o(1)} = -\kappa\frac{\partial T}{\partial x_n} + T\left(\frac{\xi}{D_{ei}} + \frac{5}{2}\frac{1-\sigma^2}{m_e}\right)\pi_{en}^{o(1)} \tag{34}$$

where quantity d_n is defined by formula (with taking into account, that $\pi_{en}^{o(1)} + \pi_{in}^{o(1)} = 0$).

$$d_n = \frac{n_e n_i\left(m_i - m_e\right)}{\rho n}\frac{\partial\ln T}{\partial x_n} + \frac{n_e n_i m_i}{\rho n}\left(\frac{\partial\ln n_e}{\partial x_n} - \sigma^2\frac{\partial\ln n_i}{\partial x_n}\right). \tag{35}$$

Here η, ζ are shear and bulk viscosity, D_a^T, D_{ab}^T are thermal diffusion and diffusion coefficients, κ is thermal conductivity, ξ is an additional kinetic coefficient.

Using definitions (34), (35) and formulas (25), (26), (32), we obtain the following expressions for the kinetic coefficients

$$D_e^T = -D_i^T = \frac{45 n_e m_e T^2}{2^{9/2}\left(z^2 n_i + \sqrt{2} n_e\right)} \lambda + O(\sigma),$$

$$D_{ei} = -D_{ie} = \frac{3T^2\left(4\sqrt{2} n_e + 13 z^2 n_i\right)}{2^{9/2} n z^4 \left(n_i + \sqrt{2} z^2 n_e\right)} \lambda + O(\sigma),$$

$$\kappa = \frac{75 n_e T^2}{2^{5/2}\left(4\sqrt{2} n_e + 13 z^2 n_i\right)} \lambda + O(\sigma), \quad \eta = \frac{5 m_e T^2}{2^3 z^4} \lambda \sigma^{-1} + O(\sigma^0), \quad \zeta = 0,$$

$$\xi = \frac{45 n_i n_e T^2}{2^{9/2} n_e m_e n \left(z^2 n_i + \sqrt{2} n_e\right)} \lambda + O(\sigma), \qquad (36)$$

which define dissipative fluxes in the system.

Conclusions

The hydrodynamics of fully ionized two-component plasma with equal component temperatures and macroscopic component velocities has been investigated taking into account that the electron-to-ion mass ratio is small. The distribution functions of the plasma components are found up to the first order in gradients of hydrodynamic variables. The kinetic coefficients of the system have been calculated.

The considered hydrodynamic states are the states in which the relaxation of the component velocities and temperatures is finished. The obtained results are not only important themselves, but they are also very important for the relaxation processes investigation at the end of relaxation. In the last situation the results obtained in the present paper give the leading order approximation for the case of small differences of component temperatures and velocities. The developed here hydrodynamics will be used in another paper for investigation of the plasma modes taking into account the relaxation.

References

1. **Landau L.D.** Kinetic equation in the Coulomb interaction case / L.D. Landau // JETP. – 1937. - V. 7. – P. 203-209 (*in Russian*).

2. **Ishimaru S.** Basic Principles of Plasma Physics/ S. Ishimaru, London, Benjamin, 1973. – 324 p.

3. **Alexandrov A.F.** Fundamentals of plasma electrodynamics /A. F. Alexandrov, L. S. Bogdankevich, A.A. Rukhadze, Moscow, Vyshchaya Shkola, 1988, 424 p. *(in Russian).*

4. **Bobylev A.V.** Relaxation of two-temperature plasma /A. V. Bobylev, I. F. Potapenko, P.H. Sakanaka // Phys. Rev. E. – 1997. – V.56, No.2. – P. 2081-2093.

5. **Akhiezer A.I.** Methods of Statistical Physics/ A.I. Akhiezer, S. V. Peletminsky, Oxford, Pergamon Press, 1981. – 376 p.

6. **Silin V.P.** Introduction to the kinetic theory of gases / V.P. Silin, Moscow, 1998, 339 p. (*in Russian*).

7. **Ferziger J.H.** Mathematical theory of transport processes in gases / J.H. Ferziger, H.G Kaper, North Holland Publishing, Amsterdam, 1972. – 579 p.

8. **Lifshitz E.M.** Physical Kinetics/ E.M. Lifshitz, L. P. Pitaevsky, Oxford, Pergamon Press, 1981. – 452 p.

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