УДК 621.396

M. V. Andreev, V. F. Borulko, O. O. Drobakhin, and D. V. Sidorov

Oles Honchar Dnipropetrovsk National University

WIDE-BAND REFLECTION COEFFICIENT MEASUREMENTS USING DIRECT AND INVERSE FREQUENCY-TIME DOMAIN TRANSFORMATIONS

Розглянуто проблему вимірювання коефіцієнту відбиття в широкій смузі частот за наявності додаткового відбиття, обумовленого застосуванням рупорні антени. Оскільки відбиття від опорної неоднорідності мало у випадку використання рупорні антени, квадрат модуля коефіцієнту відбиття визначався за автокореляційною функцією структури. Проведено порівняння методу Проні і методу виокремлення вікном автокореляційної функції шаруватої структури у часовій області для відновлення квадрата модуля коефіцієнту відбиття в широкій смузі частот. В якості досліджуваних структур були обрані навантаження з монотонно зростаючим коефіцієнтом відбиття, і шарувата структура з невеликою кількістю шарів, що має періодично осцилюючу частотну залежність коефіцієнту відбиття. Розраховані значення помилки відновлення розглянутих методів в заданій смузі частот. Продемонстровано перевагу застосування методу Проні ковзаючим вікном, як при чисельному моделюванні, так і при обробці результатів реального експерименту. Виміри проводилися у вільному просторі, в якості досліджуваної структури було обрано плоскопаралельний шар кераміки СТ5 з відомою геометричною товщиною. Запропоновано методику екстраполяції даних на краях частотного діапазону при застосуванні методу Проні. Для цього використовувалися параметри, отримані для вікон при застосуванні Проні - інтерполяції на краях діапазону.

Ключові слова: метод Проні, синтезування обвідної радіоімпульсу, шарувата структура, коефіцієнт відбиття, оптична товщина, перетворення Фур’є, автокореляційна функція.

Рассмотрена проблема измерения коэффициента отражения в широкой полосе частот при наличии дополнительного отражения, обусловленного применением рупорной антенны. Поскольку отражение от опорной неоднородности мало в случае использования рупорной антенны, квадрат модуля коэффициента отражения определялся по автокорреляционной функции структуры. Проведено сравнение метода Прони и метода выделения окном автокорреляционной функции исследуемой структуры во временной области для восстановления квадрата модуля коэффициента отражения в широкой полосе частот. В качестве исследуемых структур были выбраны нагрузка с монотонно возрастающим коэффициентом отражения, и слоистая структура с небольшим количеством слоев, имеющая периодическую осциллирующую частотную зависимость коэффициента отражения. Рассчитаны значения ошибки восстановления рассматриваемых методов в заданной полосе частот. Продемонстрировано преимущество применения метода Прони скользящим окном, как при численном моделировании, так и при обработке результатов реального эксперимента. Измерения проводились в свободном пространстве, в качестве исследуемой структуры был выбран плоскопараллельный слой керамики СТ5 с известной геометрической толщиной. Предложена методика экстраполяции данных на краях частотного диапазона при применении метода Прони. Для этого использовались параметры, полученные для окон при применении Прони - интерполяции на краях диапазона.

Ключевые слова: метод Прони, синтезирование огибающей радиоимпульса, слоистая структура, коэффициент отражения, оптическая толщина, преобразование Фурье, автокорреляционная функция.

The problem of measurement of modulus of the reflection coefficient (RC) in wide frequency band in presence of additional reflections such as reflections in a horn antenna is considered. Because the reflection from reference inhomogeneity is low in the case of horn antenna, the square modulus of the reflection coefficient was determined from the autocorrelation function of the structure. The Prony’s method and the method of separation of autocorrelation function of layered structure by windowing in time domain for the restoration of the square modulus of the RC in wide frequency band were compared. As the investigated structures have been chosen load with a monotonically increasing reflection coefficient and a layered structure with a small number of layers, which has periodically oscillating frequency dependence of the reflection coefficient.

The values of the error of restoration methods under consideration in the chosen frequency band have been calculated. It has been shown the advantage of the Prony’s method application in sliding frame by the results of the numerical simulations and real measurement data processing. The measurements were performed in free space; as the investigated structure was chosen plane-parallel layer ceramic ST5 with a known geometrical thickness. The technique of data extrapolation at the edges of the frequency band for applying Prony’s method is proposed. The parameters obtained for the windows in the application of Prony - interpolation on the edges of the frequency band were used for extrapolation.

Key words: Prony’s method, synthesis of the radiopulse envelope, layered structure, reflection coefficient, optical thickness, Fourier transform, autocorrelation function.

Introduction

Modern technologies require wide use of layered polymeric and composite materials. Due to the fact that it is necessary to obtain the desired electrical or mechanical properties of the product and ensure quality testing during manufacture it is necessary to solve both the direct and inverse problems. We will assume priority of the inverse problem of determining the parameters of the structure using the frequency characteristic of the RC. In this case, the frequency dependence of the RC must be known with high accuracy.

The need to measure the complex RC requires the use of vector network analyzers, which are bulky expensive devices, besides calibration of them is difficult because of the presence in the measuring channel coaxial-waveguide transition. The method of synthesis of the envelope of the radiopulse (MSERP) [1, 2] restores the complex RC of the structure from the square modulus of the sum of the RC and reference inhomogeneity by Fourier transform of the result of windowing in the time domain corresponding cross-correlation function (CCF) of signals of the structure and reference inhomogeneity. This method based on the concept of analytic signal uses cheaper scalar reflectometers originally based on the waveguide elements. However, applying MSERP under lack of separation of the CCF through the limited bandwidth of the measurement and overlapping the edges of windows and the informative signals implies distortion of restored complex dependence RC of the structure at the edges of the band. A way to improve accuracy is extrapolation of the measured frequency dependence to zero frequency for decrease of duration of the time signal peak in order to eliminate overlap [3].

In this paper, we propose to use the Prony's method [4; 5] for recovery frequency dependence of the square modulus of the reflection coefficient (SMRC) measured in free space. This approach avoids the data loss at the edges of the frequency band through the Gibbs phenomenon under use of the Fourier transform. The advantages of the Prony's method application are demonstrated by numerical simulations and real data processing.

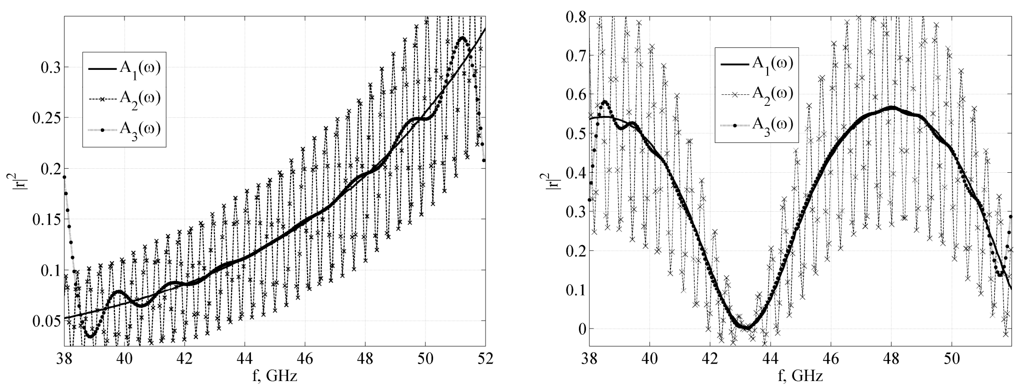
Methods SMRC Reconstruction

Let us consider the method of measuring the complex RC in free space. In the case when r_0 is the RC of reference inhomogeneity, t_0 is the time interval corresponding to the distance between the reference inhomogeneity and the structure, the signal at the output of the detector with a quadratic characteristic, can be written in following way [1]:

$$A(\omega)/k = |r_0|^2 + r_0^* R(\omega)\exp(-j\omega t_0) + r_0 R^*(\omega)\exp(j\omega t_0) + |R(\omega)|^2, \qquad (1)$$

$R(\omega)$ is the reflection coefficient of the structure, k is the factor, the symbol "*" denote complex conjugation. The term $|R(\omega)|^2$ in the right side of (1) is the spectrum of the autocorrelation function (ACF) of the response of the structure; $r_0^* R(\omega)\exp(-j\omega t_0)$ contains the required information and determines CCF response of the structure and reference inhomogeneity. According to the MSERP, the desired signal can be extracted in the time domain by window under condition that the corresponding CCF and ACF belong to different time intervals, and do not overlap with each other [1; 2].

In the numerical experiments several variants were modeled. In the first case the initial frequency dependence was the sum of the reflection from reference frequency independent inhomogeneity of 0.1 and reflection from the investigated structure with a monotonically increasing frequency characteristic $A_1(\omega)/k = |0.2\exp(j\omega t_{01}+\alpha\omega) - 0.1|^2$. The t_{01} corresponds to twice travel time of distance from the reference inhomogeneity to the investigated structure. In another case, frequency independent reference inhomogeneity and a simple three-layer structure $A_2(\omega)/k = |0.2 + R(\omega)\exp(j\omega t_{02})|^2$ are considered. The geometrical thickness of the first and the last layers of polystyrene is $d_{1,3} = 1\ mm$, and their permittivity $\varepsilon_{1,3} = 2.65$. Central air layer has the thickness $d_2 = 14\ mm$ and permittivity $\varepsilon_2 = 1$. The distance from the reference inhomogeneity to the structure $d_{02} = 350\ mm$. The size of the rectangular window, which separates the ACF in the first case was $60\ mm$ in the second case $140\ mm$, both allowed us to obtain the frequency dependence of the SMRC of the simple structure (Fig. 1, *a*) and a layered one (Fig. 1, *b*).



a) **b)**

Fig. 1. The square modulus of the sum of the RC of the reference inhomogeneity and investigated load: simple structure (case *a*) and three-layer structure (case *b*) $A_2(\omega)$. Restored SMRC by the Prony's method $A_1(\omega)$ and Fourier method $A_3(\omega)$ against frequency

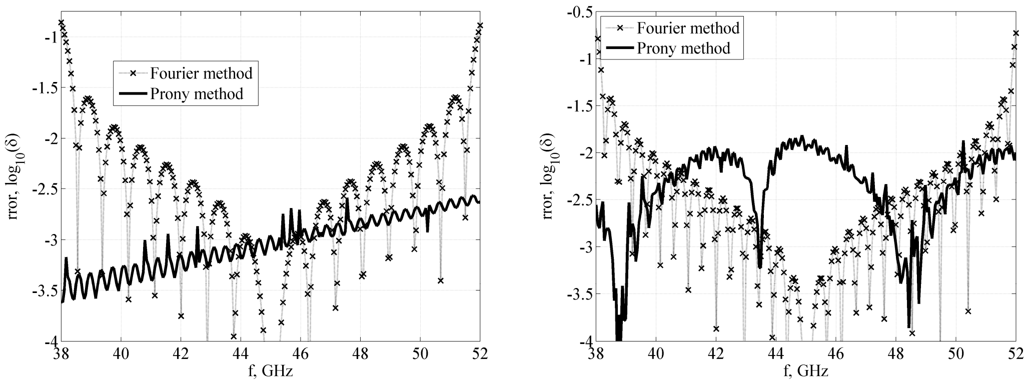
As we can see from Fig. 1, *a* and Fig. 1, *b*, lines $A_3(\omega)$ reconstructed SMRC have two peaks at the edges of the band through the Gibbs phenomenon. It is clear that the amplitude of these peaks can not be reduced, but the width of peaks can be done narrower. It requires increasing the length of the window in the time domain, but even with a good separation of peaks of the time signal overlapping ACF and CCF exists, thus in this case also there are some false samples.

In practice, to obtain time signals by spectral analysis of reflectance measurements at many frequencies parametric spectral analysis [4; 5] can be used instead of discrete Fourier transform algorithm.

The most physically correct is the application of Prony's method [5], which is a numerical algorithm for determination of complex exponents and the corresponding amplitude factors for a dependence represented as a sum of exponential components if its equidistant samples are known. The frequency dependence of the RC of a layered structure satisfies specified requirements [1; 2].

We used the Prony's method in sliding frame, the order was $M = 3$, while in the case of the simple structure (Fig. 1, *a*) was chosen frame of $n = 9$ samples (total number of samples was $N = 256$ in the frequency band $\omega \in [38; 52]\ GHz$). The value of the SMRC corresponds to the value of amplitude of the spectral components with zero exponent. For the three-layer structure (Fig. 1, *b*), the frame size should be increased to $n = 11$ due to the more complicated frequency dependence.

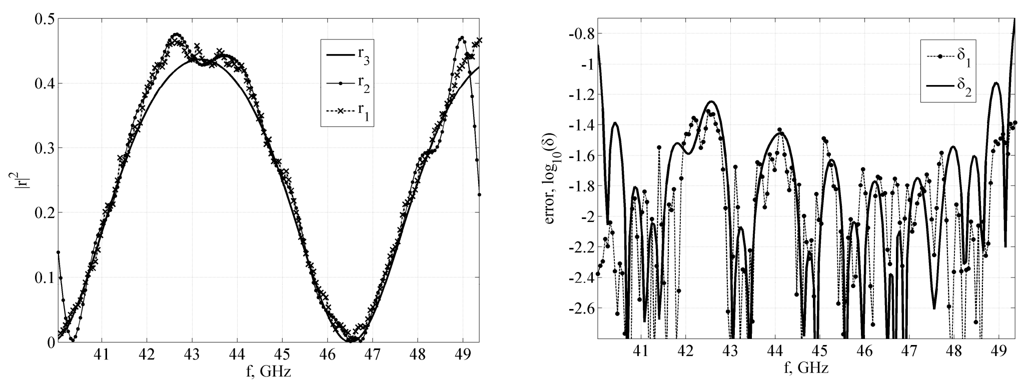
Fig. 2, *a* and Fig. 2, *b* show the frequency dependence of the absolute error for the structure with monotonous frequency dependence and for the three-layer structure, respectively. The error of the Prony's method is more uniform in the given frequency band (Fig. 2, *a*), and also much smaller ($\lg\delta \approx -2.5$) in absolute value in comparison with the discrete Fourier transform ($\lg\delta \approx -0.85$). The oscillation frequency of error (Fig. 2, *b*) is higher and the width of the first peak is less than in the previous case due to the longer duration of the window.



a) **b)**

Fig. 2. The absolute error SMRC recovery for simple structure (case *a*) and for three-layer constructions (case *b*) by the Prony's method (solid line) and by windowing in the time domain (dotted line)

In a real experiment the SMRC of a plane-parallel ceramic ST5 plate with thickness $d = 10.2$ *mm* and the permittivity $\varepsilon = 4.9$ was restored. The deviation of the reconstructed SMRC by Prony's method (r_1 in Fig. 3, *a*) from the true value (r_3 in Fig. 3, *a*) at the edges of the frequency range is less (δ_1 in Fig. 3, *b*) than for the application of the Fourier transform (r_2 in Fig. 3, *a* and δ_2 in Fig. 3, *b*).



a) **b)**

Fig. 3. The modeled r_3 and restored by the Prony's method r_1 and Fourier method r_2 SMRC of ceramic ST5 against frequency (case *a*). The absolute error recovery SMRC of ceramic ST5 (case *b*) by the Prony's method (solid line) and by windowing in the time domain (dotted line)

Conclusion

Application of the Prony's method can recover SMRC more accurately for simple and multilayer structures in comparison with the discrete Fourier transform, as the application of the model in the form of a sum of exponentials with complex exponents and amplitudes is the most physically caused to layered structures.

References

1. **Borulko V.F.** Synthesis of time signals from the square modulus of the reflection coefficient as a means of estimation of the parameters of layered dielectric structures / V. F. Borulko, O. O. Drobakhin // Soviet Non-destructive Testing. – 1986. – № 4. – P. 36–42.

2. **Drobakhin O. O.** Implementation of the method of synthesis envelope of the radio pulse according amplitude measurements using horn antenna / O. O. Drobakhin // Russ. Non-destructive Testing. – 1999. – № 7. – P. 67–74.

3. **Antropov O. S.** Improvement of the method of Fourier holography in the microwave range, based on extrapolation using the method of minimum duration / O. S. Antropov, V. F. Borulko, S. M. Vovk, O. O. Drobakhin // Radiophysics and radioastronomy. – 2010. – V. 15, № 1. – P. 89–97.

4. **Marple S. L.** Digital Spectral Analysis / S. L. Marple // Englewood Cliffs, NJ: Prentice Hall, 1987.

5. **Drobakhin O. O.** Application of the Prony method for thickness measurement of layered dielectric / O. O. Drobakhin, V. G. Korotkaya // Soviet Non-destructive Testing. – 1987. – № 5. – P. 19–30.

Надійшла до редколегії 16.07.12.